B.Sc./5th Sem (H)/PHS/22(CBCS)

2022

5th Semester Examination

PHYSICS (Honours)

Paper : DSE 1-T

[CBCS]

Full Marks : 40 | Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

[Advanced Mathematical Physics - I]

Group - A

1. Answer any *five* of the following questions : 2×5=10

(i) A symmetric tensor of rank 2 in 6-dimensional space has at most __?__ independent component. Fill in the blank.

(ii) Find the Laplace transform of $f(t)=t\sin(bt)$, where b is a constant.

(iii) Show that the Legendre's polynomials $P_0(x)=1, P_1(x)=x,$ and $P_2(x)=\frac{3}{2}x^2-\frac{1}{2}$ can form a vector space in dimension 3.

(iv) (2 Marks) For any arbitrary vector, **p** it is known that $A_{ij}\mathbf{p}$ is a covariant vector. Show that A_{ij} is a covariant tensor of rank 2.

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(v) Find the convolution $f * g$ where $f(t) = e^{-t}$ and $g(t) = \sin t$.

(vi) Find the transformation matrix corresponding to the rotation of the coordinate system $[(x_1, x_2) \to (x_1', x_2')]$ about an angle θ with respect to x_3 axis.

(vii) Explain : Covariant, Contravariant and Mixed tensors.

(viii) Write $\vec{\nabla}.(\vec{\nabla} \times \vec{A})$ and $\vec{\nabla} \times \vec{\nabla}\phi$ in tensor notation.

Group - B

2. Answer any ***four*** of the following questions : 5×4=20

(i) (a) Show that ϵ_{ijk} is the only third order isotropic tensor.

(b) Define the Christoffel symbol of first kind. 4+1=5

(ii) (a) Show that $\sum_{pq} \epsilon_{ipq} \epsilon_{jpq} = 2\delta_{ij}$ and $\sum_{ijk} \epsilon_{ijk} \epsilon_{ijk} = 6$

(b) Define direct product between two matrices. 4+1=5

(iii) Show that the map $d : P_3 \to P_3$ given by $d(f) = 3ax^2 + 2bx + c$ is a homomorphism.

(iv) (a) Show that Laplace transform of $t^n f(t)$ is $(-1)^n \frac{d^n}{ds^n} F(s)$.

(b) If $L[f(t)] = F(s)$ then $L[f(t-T)] = ?$ 4+1=5

(v) Determine the values of (i) $\delta_{ij}\delta_{ij}$, (ii) $\epsilon_{pqr} \epsilon_{pqr}$.

(vi) (a) Show that an operator $\hat{A} = -i\frac{d}{dx}$ is self adjoint in the Hilbert space (i.e. L^2 on $(-\infty, \infty)$. But it is not self adjoint for L^2 space (−1, 1).

(b) Show that eigenvalues of hermitial operators are real. 3+2=5

(vii) Calculate the element g_{rj} for the metric tensor for cylindrical polar coordinates. Hence find the square of the infinitisimal arc length $(ds)^2$ and volume dv for this coordinate system. 5

(viii) Show that the covariant derivative of a covariant vector is given by $A_{i;j} = \frac{\partial A_i}{\partial x^j} - A_K \Gamma^K_{ij}$

Group - C

3. Answer any ***one*** of the following questions : 10×1=10

(i) (a) For the vectors in three dimension

$v_1 = \hat{x} + \hat{y}, v_2 = \hat{y} + \hat{z}, v_3 = \hat{z} + \hat{x}$,

use the Gram-Schmidt orthogonalization process to create an orthonormal basis starting with v_1.

P.T.O.

(b) Consider the change of basis denoted by $x = Sx'$ where S is the transformation matrix associated with change of basis. Now consider the operator equation $y = Ax$. Show that under linear transformation A transforms as $A' = S^{-1}AS$. 5+5=10

(ii) (a) Prove the convolution theorem in Laplace transform i.e. $L\left(f(t)*g(t)\right) = F(s)*G(s)$.

(b) Show that $\Gamma^{m}_{jk} = \Gamma^{m}_{kj}$.

(c) Show that for the metric tensor $g_{ij;k} = g^{ij}_{jk} = 0$. 4+4+2=10

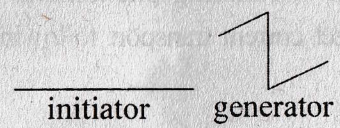
OR

[Applied Dynamics Theory]

Group - A

Answer any ***five*** of the following questions : 2×5=10

1. Draw the phase portrait for the system : $\dot{x} = \sin x$. 2
2. Discuss the stability character of the fixed points for the system $\dot{x} = x(1-x)$ using the concept of flow. 2
3. Sketch the phase diagram in the neighborhood of the equilibrium points for the system represented as : $\dot{x} + x\,\mathrm{sgn}(x) = 0$. 2
4. What is a Sinai billiard? 2
5. What do you mean by "Chaos"? How can it be demonstrated using billiards? 2
6. Using the initiator and generator shown, draw the next two stages of the iterated fractal. 2



7. Write down Navier Stokes equation. 2
8. Give two examples of Non-Newtonian fluids. 2

P.T.O.

Group - B

Answer any ***four*** of the following questions : 5×4=20

1. What is the Continuum Hypothesis in fluid dynamics? State the Newton's law of viscosity. What is the definition of fluid? 2+2+1

2. What is flow dimensionality? What is uniform and non-uniform flow? Explain. 2+3

3. Obtain the torque balance equation of the chaotic waterwheel. 5

4. Consider a population of fish in a pond of fixed size with limited food resource. Describe how the population growth is affected if (a) the start value changes and (b) the growth rate changes. 3+2

5. Discuss how scattering phenomena inside mesoscopic system affect current transport following a chaotic pattern. 5

6. Consider the following familiar system in polar coordinates : $\dot{r} = r(1-r^2), \dot{\theta} = 1$. Let D be the disk given by $x^2 + y^2 \leq 1$. Is D attractor? If not, explain why and find its basin of attraction. 5

Group - C

Answer any ***one*** of the following questions : 10×1=10

1. Consider the logistic map $x_{n+1} = rx_n(1-x_n)$ for $0 \leq x_n \leq 1$ and $0 \leq r \leq 4$. Find all the fixed points and determine their stability. Show that the logistic map has a 2-cycle for all $r > 3$. Show that the 2-cycle is stable for $3 < r < 1+\sqrt{6}$. 3+5+2

2. Discuss the features of 1D shear flow, 2D shear flow over a step and 3D shear flow in a box. What are separated and unseparated flows? Explain with proper examples. 2½×4

P.T.O.

OR

[Atmospheric Physics]

Group - A

Answer any ***five*** of the following questions : $2\times5=10$

1. (a) What is Greenhouse effect?

(b) Explain Principles of radiometry.

(c) State and explain Bouguert-Lambert Law.

(d) Explain how cyclones are produced.

(e) What do you mean by wave dispersion?

(f) What are fundamental forces of nature?

(g) Give basic principle of 'RADAR'.

(h) Distinguish between circulation and Vorticity.

Group - B

Answer any ***four*** of the following questions : $5\times4=20$

2. (a) Derive the equation for propagation of atmospheric gravity waves in a non-homogeneous medium.

(b) Give a brief note on thermal & structure of Earth's atmosphere.

(c) Give brief notes on atmospheric oscillations, quasi-biennial oscillations.

(d) Explain briefly how LIDAR can be used to study atmospheric phenomenon.

(e) Give a brief note on spectral distribution of solar radiation.

(f) Give a brief note on optical phenomenon in atmosphere.

Group - C

Answer any ***one*** of the following questions :
$10\times1=10$

3. (a) What is Rossby Waves? Describe its propagation in 3-dimension and in shared flow.

(b) Derive 'RADAR' Range equation, write a short note on various types of atmospheric 'RADAR'S. with their applications to study atmospheric phenomena.

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OR

(Classical Dynamics)

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Attempt any ***ten*** : $2\times10=20$

1. A rod of length L_0 moves with speed v along horizontal direction. The rod makes an angle θ_0 with respect to the x' axis. Determine the angle θ the rod makes with the x axis. 2

2. Consider two masses m_1 and m_2 moving in three dimensions, which are attracted to each other gravitationally, and are also acted on by a uniform gravitational field with acceleration strength g in the $-\hat{z}$ direction. Write down the Lagrangian. (Hint : You may use the concept of reduced mass and polar coordinates.) 2

3. What is the physical significance of Laplace-Runge-Lenz vector? 2

4. Two events separated by time-like interval cannot be causally related. *True* or *False*? 2

5. Show that, $E^2 = p^2c^2 + m^2c^4$ (symbols hold their usual meaning). 2

6. Two powerless rockets are heading towards each other on a collision course. As measured by Liz, a stationary Earth observer, Rocket 1 has speed 0.800c, Rocket 2 has speed 0.600c, both rockets are 50.0 m in length, and they are initially 2.52 Tm apart. ($1Tm = 10^{12}m$). What is the length of each rocket as observed by a stationary observer in the other rocket? 2

7. Atwood's machine consists of two blocks of mass m_1 and m_2 attached by an inextensible cord which suspends them from a pulley of moment of inertia I with frictionless bearings. Write down the Lagrangian of the system. 2

8. The production of Higgs particles at the LHC was achieved through the collision of two collinear beams of protons. We consider the collisions in the rest frame K of the LHC. We denote by $M=125\ GeV/C^2$ the mass of the Higgs particle, and by $m = 938\ MeV/c^2$ the mass of a proton. Suppose two protons move along the x-axis with equal but opposite velocities to produce a Higgs particle at rest upon collision. Find the required energy and velocity of the protons. 2

9. A uniform chain of length *l* and mass *m* lies on top of a triangle block of mass 2m with two equal sides of

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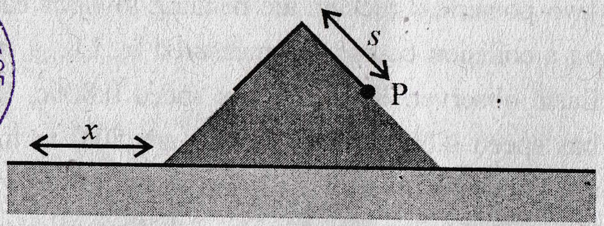
length ≥ 1 and the top corner forming a right angle. The block is free to slide on a frictionless horizontal plane.



At the right end P of the chain, a mass m is attached. The system is subjected to the gravity force and we ignore friction. At time $t = 0$ it is at rest with a length $s(0) = 1/2$ of chain lying from the right side of the block. Write down the Lagrangian of the system. 2

10. Two clocks are positioned at the ends of a train of length L (as measured in its own frame). They are synchronized in the train frame. The train travels past you at speed v. It turns out that if you observe the clocks at simultaneous times in your frame, you will see the rear clock showing a higher reading than the front clock. By how much? 2

11. The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down. *True* or *False*? 2

12. What can you do to convert a laminar flow into turbulent flow? 2

13. When a particle is constrained to move on the surface

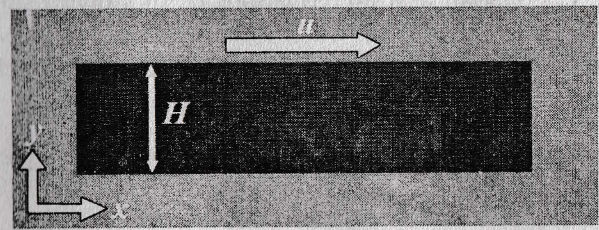
of a cylinder of radius R, the Lagrangian is given in cylindrical coordinates by :

$$L = \frac{1}{2}m\left(R^2\dot{\varphi}^2 + \dot{z}^2\right) - \frac{1}{2}k\left(z^2 + R^2\right) - mgz$$

Obtain the corresponding Hamiltonian. 2

14. Describe the physical origin of all the terms in Navier Stokes equation. 2

15. Set up the Navier Stokes equation for the Couette Flow in the steady state and fully developed flow (diagram below) : 2



Group - B

Attempt any ***four*** : 5×4=20

1. A train with proper length L moves at speed 5c/13 with respect to the ground. A ball is thrown from the back of the train to the front. The speed of the ball with respect to the train is c/3. As viewed by someone on the ground, how much time does the ball spend in the air, and how far does it travel? 5

2. Use a Minkowski diagram where the axes in frame S are orthogonal to solve the following problems :

   (a) The relative speed of S' and S is v (along the x

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direction). A meter stick lies along the x' axis and is at rest in S'. If S measures its length, what is the result?

(b) Now let the meter stick lie along the x axis and be at rest in S. If S' measures its length, what is the result? 3+2

3. Consider a mass "m" on the end of a massless rigid rod of length "L", the other end of which is free to rotate about a fixed point. This is a spherical pendulum. Find the Lagrangian and the equations of motion. 5

4. Find a differential equation for $\theta(\varphi)$ for the shortest path on the surface of a sphere between two arbitrary points on that surface, by minimizing the length of the path, assuming it to be monotone in φ. 5

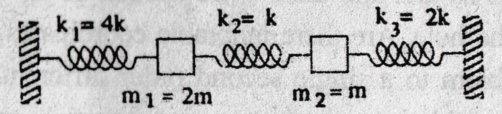
5. A particle of mass m moves in one dimension such that it has the Lagrangian as follows :

$$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 f(x) - V(x)$$

Here $f(x)$ is some differentiable function of x. Find the equation of motion and describe the physical nature of the system based on this equation. 5

6. (a) The mass flow rate through a cylindrical pipe of cross-sectional area $A = 0.5$ m^2 is $\dot{m} = 1500$ kg/s. What is the pressure difference over a distance $L = 10$ m? Viscosity of water is 1.002 milli Pa-s. 3

(b) Write down the Lagrangian for the following system : 2



Group - C

Attempt any *two* : 10×2=20

1. Consider the Navier equation ignoring the volume force, and show that :

(a) A uniform elastic material can support longitudinal waves. At what speed do they travel? 3

(b) A uniform elastic material can support transverse waves. At what speed do they travel? 3

(c) Granite has a density of 2700 kg/m^3, a bulk modulus of 4×10^{10} N/m^2 and a shear modulus of 2.5×10^{10} N/m^2. If a short spike of transverse oscillations arrives 25 seconds after a similar burst of longitudinal oscillations, how far away was the explosion that caused these waves? 4

2. A particle of mass m_1 moves in two dimensions on a frictionless horizontal table with a tiny hole in it. An inextensible massless string attached to m_1 goes through the hole and is connected to another particle of mass m_2, which moves vertically only. Give a full set of

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generalized unconstrained coordinates and write the Lagrangian in terms of these. Assume the string always remains taut and there is no friction of the string sliding at the hole. Are there ignorable coordinates? Reduce the problem to a single second order differential equation. Show this is equivalent to single particle motion in one dimension with a potential $V(r)$, and find $V(r)$.

3+2+3+2

3. Find out the eigenfrequencies of oscillation of a linear triatomic molecule. Discuss the physical implications of zero eigen frequency. 6+4

4. An object moves at speed $v_1 / c = \beta_1$ with respect to S_1, which moves at speed β_2 with respect to S_2, which moves at speed β_3 with respect to S_3 and so on, until finally S_{N-1} moves at speed β_N with respect to S_N (See Figure). Show by induction that :

$\beta_N = \dfrac{P_N^+ - P_N^-}{P_N^+ + P_N^-}$ where $P_N^+ = \Pi_{i=1}^N (1+\beta_i)$ and

$P_N^- = \Pi_{i=1}^N (1-\beta_i)$ 10