

2022

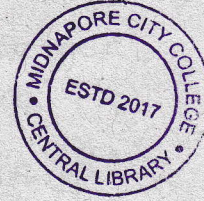
3rd Semester Examination

PHYSICS (Honours)

Paper : C 5-T

[Mathematical Physics - II]

[CBCS]



Full Marks : 40

Time : Two Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

**Group - A**

1. Answer any *five* of the following : 2×5=10

- (a) Define regular and apparent singular point.
- (b) What are Dirichlet conditions for a function to be piece-wise regular in a given interval ?
- (c) Write down the generating relation of Bessel function and show that  $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$ .
- (d) Define cyclic coordinates. Show that the generalized momentum conjugate to cyclic coordinate is conserved.

P.T.O.

( 2 )

- (e) Write down the Hermite's polynomial and hence show that  $H'_n(x) = 2nH_{n-1}(x)$ .
- (f) Write down the Laplace's equation in spherical polar coordinates.
- (g) Prove that  $\Gamma(n+1) = n\Gamma(n)$ ,  $n > 0$ .
- (h) Derive the canonical equations of Hamiltonian.

**Group - B**

2. Answer any *four* of the following: 5×4=20

- (a) Express  $f(x) = x^2$  as a Fourier's series in the interval  $-\pi \leq x \leq \pi$ , hence show that at  $x = \pi$ ,  $\sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ . 4+1
- (b) Prove that the Legendre's polynomials  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ , symbols have their usual meaning. 5
- (c) Write down the Euler's equation in variational problems and using this show that the path of shortest (brachistochron) time (chronos) of a particle is a cycloid. 1+4

( 3 )

- (d) Write down the Laplace's equation on a plane in terms of the polar coordinates. Solve it by the method of separation of variables and write the general expression of the solution which is finite at  $r = 0$  and single valued in  $\theta$ . 1+4
- (e) A simple pendulum consists of mass  $m_2$ , with a mass  $m_1$  at the point of support which can move on a horizontal line in the plane in which  $m_2$  moves. Find the Lagrangian of the system and Lagrange's equations. 3+2

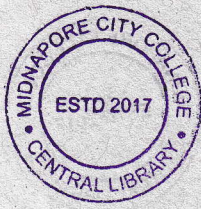
- (f) A bar of length  $L$  whose entire surface is insulated including its ends at  $x = 0$  and  $x = L$  has initial temperature  $f(x)$ . Determine the subsequent temperature of the bar. 5

**Group - C**

3. Answer any *one* of the following: 10×1=10

- (a) (i) State Hamilton's principle and derive Lagrange's equation of motion from it. Discuss how the result will be modified for non-conservative forces.
- (ii) From the generating function of Legendre's polynomials, show that  $nP'_n(x) = xP'_n(x) - P'_{n-1}(x)$ .
- (iii) State the Parseval's identity of Fourier series. (1+4+1)+2+2

P.T.O.



( 4 )

- (b) (i) Expand  $f(x) = \sin x$ ,  $0 < x < \pi$ , in a Fourier cosine series.
- (ii) Evaluate the value of integration  $\int_0^1 x^4 (1-x)^3 dx$ , using Beta function.
- (iii) Write down the two dimensional wave equation in polar coordinates and solve it to find the eigen values and eigen functions in case of a circular membrane. 4+2+4