B.Sc/1st Sem (H)/PHS/22(CBCS)

2022

1st Semester Examination

PHYSICS (Honours)

Paper : C 1-T

(Mathematical Physics I)

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *five* questions: 2×5=10

1. Plot $f(x) = \log_{10}(x), \log_5(x)$ and $\log_2(x)$ in a single graph.

2. Find the Maclaurin's series of the function $\{\log(1-x)\}$ using the definition of Maclaurin series, (extended to infinity).

3. If the magnitude of a vector $\vec{A}$ is constant with respect to time, show that $\frac{d\vec{A}}{dt}$ is perpendicular to $\vec{A}$.

4. Evaluate $\iint_S \vec{r}.\hat{n}ds$ for a closed surface S.

P.T.O.

5. Using Dirac Delta function properties,

   Find (i) $\int_0^2 (4x+1)\delta(x+1)\,dx$ (ii) $\int_{-\infty}^{\infty} t^2\delta(3t-6)\,dt$

6. If $f(x,y)=e^{-x}\sin(x+y)$, check whether the expression, $\frac{\partial f}{\partial y}-\frac{\partial f}{\partial x}=f(x,y)$ is correct or not.

7. A regular deck of cards has 52 cards. Assuming that you do not replace the card you had drawn before the next draw, what is the probability of drawing three aces in a row?

8. Find the general solution of the differential equation $\frac{dy}{dx}+y=\sin x$.

Group - B

Answer any ***four*** questions : 5×4=20

9. Check Stoke's theorem for the vector $\vec{V}=(2xz+3y^2)\hat{j}+(4yz^2)\hat{k}$ for the surface of a unit square located at first quadrant of *Y-Z* plane.

10. (a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, –1, 2).

    (b) Find the area of both loops of the lemniscate, $\rho^2=a^2\cos 2\phi$.

    (c) Explain the significance of a solenoidal vector.

    2+2+1

11. (a) Solve $\frac{dy}{dx}+xy=xy^2$.

    (b) Represent $\vec{A}=2y\hat{i}-z\hat{j}+3x\hat{k}$ in cylindrical co-ordinates. 2½+2½

12. (a) Find the Wronskian of the functions :

    (i) $y_1(t)=\sin(t)$ and $y_2(t)=2\sin(t)$.

    (ii) $y_1(t)=\sin(t)$ and $y_2(t)=t\sin(t)$. Comment on whether y_1 and y_2 are linearly independent or dependent. 3

    (b) Define conditional probability and independence. 2

13. (a) Show that $\delta(kx)=\frac{\delta(x)}{|k|}$, where k is any (nonzero constant).

    (b) Prove $\iiint_V \vec{\nabla}\times\vec{B}\,dV=\iint_s \hat{n}\times\vec{B}\,dS$. 2½+2½

14. (a) A store contains 35 light green parakeets (14 females and 21 males) and 44 sky blue parakeets (28 females and 16 males). You randomly choose one from the parakeets. What is the probability that it is a female on sky blue parakeet?

    (b) A magnetic field $\vec{B}$ is given by $\vec{B}=\rho^{-2}\hat{\varphi}+k\hat{z}$. Find and $\vec{\nabla}.\vec{B}$ and $\vec{\nabla}\times\vec{B}$. 3+2

P.T.O.

Group - C

Answer any *one* of the following questions :

10×1=10

15. (a) Evaluate $\iint_s \vec{A}.\hat{n}ds$ for $\vec{A}=(x+y^2)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant. 3

(b) Find the shortest distance from (6, –4, 4) to the line joining (2, 1, 2) and (3, –1, 4). 3

(c) Verify Green's theorem in the plane for $\int_c (2x-y^3)dx - xydy$, where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. 4

16. (a) Assume that each face of a six faced dice is equally likely to land uppermost. Consider a game which involves the tossing of five such dice. Calculate the probability that the number 4 appears uppermost (i) in exactly one dice, (ii) in exactly two dice, (iii) in exactly five dice, (iv) in none of five dice, (v) in at least one dice. 1+1+1+1+1

(b) In a certain factory turning razor blades, there is a small chance 1/500 for any blade to be defective. The blades are in packets of 10. Use Poisson's distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 10,000 packets. Explain Bayes' theorem. 3+2