

2022

5th Semester Examination
PHYSICS (Honours)

[Quantum Mechanics and Applications]

Paper : C 11-T

[CBCS]

Full Marks : 40

Time : Two Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

1. Answer any *five* of the following questions : $2 \times 5 = 10$

(a) Show that the momentum operator is Hermitian. 2

(b) Which one of the following wave functions is a well-behaved wave function in the range $-\infty < x < \infty$? Justify your answer.

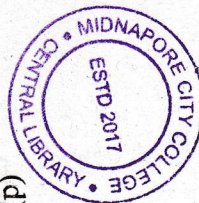
(i) $\psi_1 = A \exp(-x)$

(ii) $\psi_2 = A \exp(-x^2)$

(c) The operator $\left(x + \frac{d}{dx} \right)$ has the eigenvalue α .

Derive the corresponding eigenfunction.

P.T.O.



(2)

(d) In case of a quantum harmonic oscillator, show that zero-point energy is equal to $(1/2) \hbar\omega$ using Heisenberg's uncertainty principle. 2

(e) Plot the ground state and the first excited state wave functions of a linear harmonic oscillator. Also plot the corresponding probability density functions.

(f) Two particles (masses m_1 and m_2) are attached to the ends of a massless rigid rod of length "a". The system is free to rotate in three dimensions about the (fixed) center of mass. Find out the allowed energy levels. 2

(g) In the Stern-Gerlach experiment why is it necessary to use a beam of neutral atoms and not ions?

(h) What are the values of L, S, and J and the multiplicity of the level having spectral term $4P_{5/2}$?

Group - B

Answer any *four* from the following questions :

5×4=20

2. Set up the time-independent Schrödinger equation for the one-dimensional potential

$$V(x) = 0 \text{ for } 0 < x < L$$

= ∞ elsewhere.

Write the appropriate boundary conditions. Obtain the energy eigenvalues and the corresponding eigenfunctions.

1+2+2



(3)

3. Determine the position probability density and the probability current density for the Gaussian wave packet

$$\psi(x, 0) = A \exp\left(ikx - \frac{a^2 x^2}{2}\right)$$

2+3

4. Suppose a spin $1/2$ particle is in a state : $\chi = \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$.

What are the probabilities of getting $\hbar/2$ and $-\hbar/2$ if you measure S_x and S_z ? 5

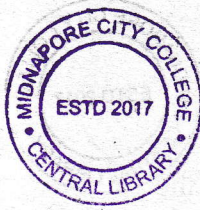
5. What is the most probable value of "r" in the ground state of Hydrogen? (Ground state is given by $R_{10}(r) = 2a^{-3/2} \exp(-r/a)$) What are the sources of l-degeneracy and m-degeneracy. 3+2

6. (a) Show that the quantum mechanical probability of finding the linear harmonic oscillator in the ground state outside the classical limits of motion, is approximately 16%. (Given, $\int_0^1 e^{-z^2} dz \approx 0.746$)

(b) What new information do we get from the quantum picture of a harmonic oscillator compared with the classical picture?

7. (a) Explain the origin of sodium D1 and D2 lines.

(b) Show that $[\hat{x}^3, p] = i\hbar 3\hat{x}^2$. 3+2



(4)

Group - C

Answer any *one* of the following questions :

10×1=10

8. A particle in the harmonic oscillator potential starts out in the state :

$$\psi(x, 0) = A[3\psi_0(x) + 4\psi_1(x)]$$

(a) Find A. 1

(b) Construct $\psi(x, t)$. 1

(c) What is the frequency of oscillation of $|\psi(x, t)|^2$? 2

(d) How the answer of "c" will change if $\psi(x, 0) = A[3\psi_0(x) + 4\psi_2(x)]$? 4

(e) What will be the average energy of the system in both the cases? 2

9. What is Larmor precession? Explain the theory of Anomalous Zeeman Effect. Why Paschen Back effect and Normal Zeeman effect have certain similarity?

3+5+2
