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B.Sc./1st Sem (H)/MATH/22 (CBCS)

2022

1st Semester Examination

MATHEMATICS (Honours)

Paper : GE-1T

(Calculus Geometry and Differential Equation)

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ***ten*** questions : 2×10=20

(a) If $y = x^{n-1}.\log x$; show that $y_n = \dfrac{(n-1)!}{x}$

(b) Show that origin is a point of inflexion on the curve $y = x\cos 2x$.

(c) Prove that the curve $y = e^x$ is convex to the x-axis at every point.

(d) Find the envelope of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$, m being parameter.

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(e) If $I_n = \int_0^{\pi/4} \tan^n x dx$, show that $I_{n+1} - I_{n-1} = \frac{1}{n}$.

(f) Find the length of the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

(g) Determine the nature of the conic presented by $9x^2 + 24xy + 16y^2 - 126x + 82y - 59 = 0$

(h) Find the polar co-ordinate of the point whose Cartesian co-ordinates are $\left(\sqrt{3}, 1\right)$.

(i) Find the equation to a sphere whose centre is (2, 5, 4) and which passes through the point $(-1, 3, 2)$.

(j) Define non-linear ODE of first order.

(k) Examine if the ODE $(1+xy)y\,dx + (1-xy)\,xdy = 0$ is exact.

(l) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(m) Find the equation of the cylinder whose generators are parallel to the line $-3x = 6y = 2z$ and whose guiding curve is the ellipse $2x^2 + y^2 = 1,\ z = 0$.

(n) Define asymptote of a curve.

(o) Through an example, explain singular solution of an ODE.

Group - B

2. Answer any ***four*** questions : 5×4=20

(a) Find the asymptotes of the curve

$$x(x-y)^2 - 3\left(x^2 - y^2\right) + 8y = 0.$$

(b) Show that the section of the hyperbolic paraboloid $\frac{x^2}{2} - \frac{z^2}{3} = y$ by the plane $3x - 3y + 4z + 2 = 0$ is a hyperbola.

(c) Solve the ODE : $\left(x^2y^3 + 2xy\right)dy = dx$.

(d) Find a reduction formula for $\int \sin^m x \cos^n x dx$, where m and n are positive integers. Use it to evaluate $\int_0^{\pi/2} \sin^8 x \cos^6 x\, dx$.

(e) Find a and b such that

$$\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$$

(f) If r_1 and r_2 be two mutually perpendicular radius vectors of the ellipse $r^2 = \frac{b^2}{1 - e^2\cos^2\theta}$, prove that $\frac{1}{r_1^2} + \frac{1}{r_2^2} = \frac{1}{a^2} + \frac{1}{b^2}$. $\left[b^2 = a^2\left(1 - e^2\right)\right]$.

Group - C

3. Answer any ***two*** questions : 10×2=20

(a) (i) Use suitable integrating factor to solve the ODE $x\,dy - y\,dx - \cos\frac{1}{x} = 0$.

(ii) Show that the area bounded by one arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ and the x-axis is $3\pi a^2$ sq. units.

(b) (i) Find the equation of a cone whose vertex is the point $p(\alpha,\beta,\gamma)$ and whose generating lines pass through the conic $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0$

If the section of this cone by the plane $x=0$ is a rectangular hyperbola, show that the locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(ii) Find the equation of a sphere that passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) and touches the plane $2x+2y-z=15$.

(c) (i) State Leibnitz's theorem for n^{th} derivative of the product of two functions. Use this to solve the following problem.

If $y = e^{a\sin^{-1}x}$, then prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-\left(n^2+a^2\right)y_n=0$$

(ii) If PM, PN be perpendiculars drawn from any point P on the curve $y=ax^3$ upon the coordinate axes, show that the envelope of MN is $27y+4ax^3=0$.

(d) (i) Find the surface area and the volume of the ellipsoid formed by the revolution of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ round its major axis.

(ii) If $J_n=\int_0^{\pi/2}\cos^n x\,dx$, show that $J_n=\frac{n-1}{n}J_{n-2}\ (n>2)$. Hence find J_9.
