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B.Sc./1st Sem (H)/MATH/22 (CBCS)

2022

1st Semester Examination

MATHEMATICS (Honours)

Paper : C 2-T

[Algebra]

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any *ten* questions : 2×10=20

(a) If a, b, c be three positive real numbers in Harmonic Progression and n be a positive integer greater than 1, then prove that $a^n + c^n = 2b^n$.

(b) Geometrically represent the complex number $z = a + b\,i$.

(c) Find the conditions that the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ are in G.P.

P.T.O.

(d) Apply Descartes' rule of signs to determine the nature of the roots of the equation $x^4+x^2+x-1=0$.

(e) Diminish the roots of $4x^3-8x^2-19x+38=0$ by 2.

(f) If $a,b\in\mathbb{Z}$, not both zero, such that $gcd(a,b) = a\ u + b\ v$, prove that $gcd\ (u,\ v) = 1$, where $u,v\in\mathbb{Z}$.

(g) Can a null vector be an element of a basis set ? Support your answer.

(h) Find the last two digits in 7^{100}.

(i) If a row echelon matrix R has r non-zero rows, then prove that rank of $R=r$.

(j) If λ be an eigen value of an $n\times n$ matrix A, prove that λ^m is an eigen value of the matrix A^m, where $m\in\mathbb{Z}^+$.

(k) Show that the subspace $U+W$ is the smallest subspace of vector space V containing the subspaces U and W.

(l) For what real values of k is the set $S=\{(k,1,1,1),(1,k,1,1),(1,1,k,1),(1,1,1,k)\}$ linearly independent in vector space $\mathbb{R}^4$?

(m) Let V and W be vector spaces over a field F, and $T:V\to W$ be a linear mapping. Prove that T is injective if and only if $Ker\ T=\{\theta\}$.

(n) Use Euclidean algorithm to find integers u and v satisfying $52u-91v=78$.

(o) Use Division algorithm to show that the cube of any integer is of the form $9k$ or $9k\pm1$, $k\in\mathbb{Z}$.

Group - B

2. Answer any ***four*** questions : 5×4=20

(a) Prove that $\arg z-\arg(-z)=\pm\pi$ according as $\arg z>0$ or $\arg z<0$.

(b) If a, b, c be positive real numbers and $abc=k^3$, prove that $(1+a)(1+b)(1+c)\ge(1+k)^3$.

(c) Show that the equation $(x-a)^3+(x-b)^3+(x-c)^3+(x-d)^3=0$, where a,b,c,d are not all equal, has only one real root.

(d) If α,β,γ be the roots of the equation $x^3+px^2+qx+r=0$, then form the equation whose roots are $\alpha+\frac{1}{\alpha}$, $\beta+\frac{1}{\beta}$, $\gamma+\frac{1}{\gamma}$.

P.T.O.

(e) Find a basis and dimension of the subspace S of $\mathbb{R}^3$ defined by
$S = \{(x, y, z) \in \mathbb{R}^3 : 2x + y - z = 0\}$.

(f) Use the principle of induction to prove that $2.7^n + 3.5^n - 5$ is divisible by 24, $\forall n \in \mathbb{N}$.

Group - C

Answer any *two* questions : 10×2=20

3. (a) If $\alpha = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ and $gcd(n, p) = 1$, then prove that $1 + \alpha^p + \alpha^{2p} + ... + \alpha^{(n-1)p} = 0$.

(b) Prove that in the euqation $f(x) = 0$ with real coefficients, imaginary roots occur in conjugate pairs.

5+5

4. (a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method.

(b) State Cayley-Hamilton theorem. Using the theorem describe a method of computing A^{-1} when A is a non-singular square matrix. 6+(1+3)

5. (a) If $\begin{pmatrix}1\\2\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$ are the eigen vectors corresponding the eigen values 1, 2, 0 of the real square matrix A of order 3, then find A.

(b) Find a linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $Im\, T$ is the subspace
$U = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ 5+5

6. (a) For what value of k the planes $x - 4y + 5z = k$, $x - y + 2z = 3$, and $2x + y + z = 0$ intersect in a line ? Find the equations of the line in that case.

(b) If $z = \cos\theta + i\sin\theta$ and $m \in \mathbb{Z}^+$, then show that
$\frac{z^{2m} - 1}{z^{2m} + 1} = i\tan m\theta$. (4+2)+4

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