B.Sc./1st Sem (H)/MATH/22 (CBCS)

2022

1st Semester Examination

MATHEMATICS (Honours)

Paper : C 1-T

[Calculus, Geometry and Differential Equation]

[CBCS]

Full Marks : 60 | Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer any ***ten*** questions : 2×10=20

   (i) Find y_n for the function $y = \dfrac{x^n}{x-1}$.

   (ii) Show that the curve $y^3 = 8x^2$ is concave to the foot of the ordinate everywhere except at the origin.

   (iii) If the axes are rotated through an angle 45° without changing the origin, then find the new form of the equation $x^2 - y^2 = a^2$.

P.T.O.

(iv) Find the equation of the circle lying on the sphere $x^2+y^2+z^2-2y-4z=11$ and having its centre at (1,3,4).

(v) Find the total area of the circle $x^2+y^2+2x=9$.

(vi) If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, for $n\geq 2$, find the value of I_n+I_{n-2}.

(vii) Find the asymptotes of the curve $x^3+y^3=3axy$.

(viii) Find the integrating factor of

$$\left(1+x^2\right)y_1+y=e^{\tan^{-1}x}.$$

(ix) Find the singular solution of $y=x\dfrac{dy}{dx}-\left(\dfrac{dy}{dx}\right)^2$.

(x) Find the nature of the conic

$$3x^2+2xy+3y^2-16x+20=0$$

(xi) Calculate the sum of the reciprocals of two perpendicular focal chord of the conic $\frac{l}{r}=1+e\cos\theta$.

(xii) Show that $\lim\limits_{x\to\infty}\left(\dfrac{ax+1}{ax-1}\right)^x=e^{2/a},\ a>0$.

(xiii) If $u=\sin ax+\cos ax$, show that

$$u_n=a^n\left\{1+(-1)^n\sin 2ax\right\}^{\frac{1}{2}}.$$

(xiv) Solve $p-\dfrac{1}{p}-\dfrac{x}{y}+\dfrac{y}{x}=0$ where $p\equiv\dfrac{dy}{dx}$.

(xv) Evaluate $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)$.

Group - B

2. Answer any ***four*** questions : 5×4=20

(i) State and prove Leibnitz's theorem. If $y=\tan^{-1}x$ find $(y_n)_0$ by using Leibnitz's theorem.

(ii) Prove that the locus of the middle points of focal chords of a conic is an another conic.

(iii) If $J_n=\int\sin n\theta\ \sec\theta\,d\theta$, show that

$$J_n+J_{n-2}=-\frac{2}{n-1}\cos(n-1)\theta.$$ Hence deduce the value $\int_0^{\pi/2}\dfrac{\sin 3\theta\ \cos 3\theta}{\cos\theta}d\theta$.

(iv) If S be the length of the arc of $3ay^2=x(x-a)^2$, measured from the origin to the point (x,y), show that $3s^2=4x^2+3y^2$.

P.T.O.

(v) Find the equation to the right circular cylinder of radius a, whose axis passes through the origins and makes equal angles with the co-ordinates axes.

(vi) Solve : $16x^2 + 2\left(\frac{dy}{dx}\right)^2 y - \left(\frac{dy}{dx}\right)^3 x = 0$.

Group - C

3. Answer any ***two*** questions : 10×2=20

(i) (a) Explain L'Hospital Rule. Using L'Hospital Rule prove that

$$\lim_{x \to \infty} \left[\frac{a_1^{1/x} + a_2^{1/x} + \ldots + a_n^{1/x}}{n} \right]^{nx} = a_1 a_2 \ldots a_n.$$

(b) Find the envelop of the straight line $\frac{x}{a} + \frac{y}{b} = 1$, a and b are variable parameters connected by the relation $a + b = c$. 5+5

(ii) (a) What is a great circle ? Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the great circle.

(b) Reduce the equation $3x^2 + 5y^2 + 3z^2 + 2yz + 2zx + 2xy - 4x - 8z + 5 = 0$, to the standard form and find the nature of the conic. 3+7

(iii) (a) Find the volume of ellipsoid generated by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about major axis and minor axis.

(b) Define singular and general solution of the differential equation. Find the both solutions of the following differential equation : $p^3 x - p^2 y - 1 = 0$. 5+5

(iv) (a) Find the rectilinear asymptotes of the following curve :

$$x^3 + x^2 y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$

(b) If $f(m,n) = \int_0^{\pi/2} \cos^m x \sin nx \, dx$ prove that

$$f(m,n) = \frac{1}{m+n} + \frac{m}{m+n} f(m-1, n-1),$$

$m, n > 0$. Hence deduce that

$$f(m,n) = \frac{1}{2^{m+1}} \left(\frac{2}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \ldots + \frac{2^m}{m} \right).$$

5+5
