2022

5th Semester Examination

MATHEMATICS (Honours)

Paper : C 11-T

[Partial Differential Equations and Applications]

[CBCS]

Full Marks : 60   Time : Three Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

Group - A

1. Answer any *ten* questions :   2×10=20
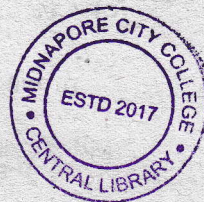
   (a) What is ballistics? Write different types of ballistics.

   (b) Define quasi-linear and semi-linear partial differential equation.

   (c) Find the general solution of second order PDE

   $$4\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0.$$

   (d) What is the nature of the second order PDE

   $$\frac{\partial^2 z}{\partial y^2} - y\frac{\partial^2 z}{\partial x^2} + x^3 z = 0?$$

                                                                P.T.O.

(2)

(e) Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then prove that the Cauchy problem $a\dfrac{\partial z}{\partial x} + b\dfrac{\partial z}{\partial y} = 1$, $x, y \in \mathbb{R}$ with $z(x, y) = x$ on $ax + by = 1$ has a unique solution.

(f) Find the characteristic curve of PDE :

$2y\dfrac{\partial z}{\partial x} + \left(2x + y^2\right)\dfrac{\partial z}{\partial y} = 0$ which is passing through the point $(0, 0)$.

(g) Find the equations of the characteristic curves of the PDE $\left(x^2 + 2y\right)\dfrac{\partial^2 z}{\partial x^2} + \left(y^3 - y + x\right)\dfrac{\partial^2 z}{\partial y^2}$
$+x^2\left(y - 1\right)\dfrac{\partial^2 z}{\partial x \partial y} + 3\dfrac{\partial^2 z}{\partial x} + z = 0$ which are passing through the point $x = 1, y = 1$.

(h) Let $z(x, t)$ be the equation of $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial t^2}$ with $z(x, 0) = \cos(5\pi x)$ and $\dfrac{\partial z}{\partial t}(x, 0) = 0$. Then prove that $z(1, 1) = 1$.

(i) Show that the solution of the PDE :

$x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 0$ is of the form $f(y/x)$.

(3)

(j) Prove that the partial differential equation $x\dfrac{\partial^2 z}{\partial x^2} + y\dfrac{\partial^2 z}{\partial y^2} = 0$ is elliptic type for $x < 0, y > 0$.

(k) Find the two families of surfaces that generate the characteristics of $(3y - 2z)\dfrac{\partial z}{\partial x} + (z - 3x)\dfrac{\partial z}{\partial x}$
$= 2x - y$.

(l) Find the partial differential equation by eliminating arbitrary constants a and b from $z = (x + a)(y + b)$.

(m) Define apsidal angle and apsidal distance.

(n) Prove that a planet has only a radial acceleration towards the Sun.

(o) Prove that at an apse on a central orbit, the velocity is proportional to the reciprocal of the radius vector.

Group - B

2. Answer any *four* questions : $5 \times 4 = 20$

(a) A particle moves with a central acceleration $\mu \div (\text{distance})^2$; it is projected with velocity v at a distance $\mathbb{R}$. Show that its path is a rectangular hyperbola if the angle of projection is $\sin^{-1}\left[\mu \middle/ \left\{\sqrt{vR\left(v^2 - \dfrac{2\mu}{R}\right)^{1/2}}\right\}\right]$.

P.T.O.

(b) A spherical raindrop falls through a cloud while accumulating mass at a rate λr^2 where r is its radius and $\lambda > 0$. Find its velocity at time t if it starts from rest with radius a.

(c) Find the integral surface of the PDE, $x(z+2a)p + (xz+2yz+2ay)q = z(z+a)$.

(d) Using the method of separation of variables, solve :

$\frac{\partial z}{\partial x} = q\frac{\partial z}{\partial t} + z$ where $z(x,0) = 6e^{-3x}$.

(e) Reduce the wave equation $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$ to canonical form.

(f) Solve $z^2 = pqxy$ by Charpit's method.

Group - C

3. Answer any *two* questions : 10×2=20

(a) (i) If a point moves on a Curve with constant tangential acceleration such that the magnitudes of the tangential velocity and normal acceleration are in a constant ratio, find the (s, ψ) equation of the curve.

(ii) Solve $(D^3 - 3DD'^2 - 2D'^3)z = \cos(x+2y)$.

(b) (i) A particle is projected with velocity V from

the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the

vertex is $2\sqrt{\dfrac{a}{g}}\tan^{-1}\left[\sqrt{\dfrac{4ag}{V}}\,\right]$. 4+6

(ii) Using the method of separation of variables, solve the following wave equation described by

PDE : $\dfrac{\partial^2 z}{\partial t^2} = 4\dfrac{\partial^2 z}{\partial x^2}$

BCS : $z(0,t) = 0,\ z(s,t) = 0$

ICS : $z(x,0) = 0,\ \left(\dfrac{\partial z}{\partial t}\right)_{t=0} = 5\sin\pi x$. 5+5

(c) (i) Solve the boundary value problem

$\dfrac{\partial u}{\partial t} = k\dfrac{\partial^2 u}{\partial x^2}$, $u(x,0) = 20$, $u(0,t) = 0$,
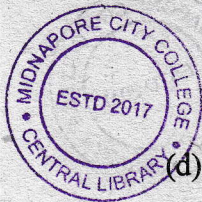
$u(L,t) = 0$.

(ii) Find the integral surface of the linear PDE

$2y(z-3)\dfrac{\partial z}{\partial x} + (2x-z)\dfrac{\partial z}{\partial y} = y(2x-3)$ which

passes through the circle $x^2 + y^2 = 2x, z = 0$. 5+5

P.T.O.

(6)

(d) (i) Solve two dimensional Laplace's equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

with BCS $z(0, y) = 0$, $z(l, y) = 0$

and $z(x, y) \to 0$ as $y \to \infty$

$$z(x, 0) = f(x)$$

(ii) Let $u(x, y)$ be the solution of the Cauchy

problem $\quad \dfrac{\partial u}{\partial y} - x\dfrac{\partial u}{\partial x} + u = 1,$ where

$-\infty < x < \infty$, $y \geq 0$ and $u(x, 0) = \sin x$, then find $u(0, 1)$. 5+5

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