বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

BCA

1st Semester Examination 2021

DISCRETE MATHEMATICS

PAPER—1103

Full Marks : 70

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Group – A

Answer any *four* questions. 4×15

1. (a) Show that the set $\{a + b\sqrt{2}; a, b \in Q\}$ where Q is the set of rational numbers, forms a group with respect to addition. 7

(b) Simplify the expression by K-map method

$$x_1x_2\bar{x}_3 + x_1x_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2x_3$$ 8

2. (a) Prove that every proper subgroup of a group of order 6 is commutative. 8

(b) Show that an integral domain that has a finite number of elements is a field. 7

3. (a) Solve the recurrance relation

$$a_r + 3a_{r-1} + 2a_{r-2} = f(r)$$

where $f(r) = \begin{cases} 1 & where\ r = 2 \\ 0 & otherwise \end{cases}$ 8

(b) Find the discrete numeric function corresponding to the generating function $A(z) = \dfrac{z^5}{5 - 6z - z^2}$. 7

4. (a) Prove that a pendant edge in a connected graph G is contained in every spanning tree of G. 7

(b) In how many arrangements of COMPUTER be arranged whether vowels are adjacent. 8

5. (a) Examine the mapping f : $z \to z$ defined by f(x) = 4x + 7, $x \in z$ is bijective or not. 8

(b) Use mathematical induction to prove that $16^n + 10n - 1$ is divisible by 25 $\forall n \geq 1$. 7

6. (a) Prove that for a simple graph with n vertices and m components can have at most (n – m) (n –m + 1) / 2 edges. 7

(b) Define Hasse diagram of a poset (S, R) where S is a non-empty set with the relation R.

Let X = {1, 2, 3, 4, 5, 6} and '/' (divided by) is a partial order relation on X. Draw the Hasse diagram on (X, /). 8

7. (a) Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology and 30 do not study any of the three subjects

(i) Find the number of students studying all three subjects. 8

(ii) Find the number of students studying exactly one of the three subjects.

(b) Prove that a graph is a tree if and only if there is a unique path between every pair of vertices in G. 7

8. (a) Prove that the roots of $x^n - 1 = 0$, $n \in z$ form a subgroup of the multiplicative group of non-zero complex numbers. Is the subgroup cylic ? 8

(b) Prove that an undirected graph possesses an Eulerian path if and only if it is connected and has either zero or two vertices of odd degree. 7

Group – B

Answer any *one* question. 1×10

9. (a) Define Euler graph with an example.

(b) For A = {a, b, {a, c}, φ} determine $\{\{a,c\}\} - A$.

(c) Construct truth table for the statement.

$p \leftrightarrow (\bar{p} \vee \bar{q})$. 3+3+4

10. (a) Let A = {3, 7, 5} and B = {4, 5, 8}. Write down total number of distinct relations from A to B.

(b) Given that A $\subseteq$ C and B $\subseteq$ D, show that

A X B $\subseteq$ C X D.

(c) A tree has 2 vertices of degree 2, one vertex of degree 3 and 3 vertices of degree 4. How many vertices of degree I does it have? 3+4+3

(Internal Assessment : 30)