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PG CBCS

M.SC. Semester-I Examination, 2021

DEPARTMENT OF PHYSICS

PAPER: PHS 101

(MATHEMATICAL PHYSICS & CLASSICAL MECHANICS)

Full Marks: 40 **Time: 2 Hours**

Write the answer for each unit in separate sheet

PHS 101.1 MATHEMATICAL PHYSICS

Answer any TWO questions of the following: 2X10=20

1. a) Discuss the difference between removable and non- removable singularity with examples.

b) Evaluate the value of the contour integral $\frac{1}{2\pi i}\oint_C \frac{e^{4z}-1}{\cosh z - 2\sinh z}dz$ around the unit circle C traversed in the anti-clockwise direction. (3+7)

2. a) The characteristic equation of a 3 x 3 matrix is $A^3 - 5A^2 - 4A - 12I = 0.$ Find out Det (A^T).

b) Consider the following matrix

$$A = \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$$

Find out the eigenvalues of the matrix $B = A^4 - 3A^3 + 3A^2 - 2A + 8I$ (4+6)

3. a) Prove that the set of vectors {(4,1,-5), (2,-3,1), (1,1,1)} is an orthogonal set of vectors in R^3 with standard inner product. Is it a basis of R^3. Give reasons.

b) Find the matrix representation of linear transformation T on $V_3(R)$ defined as

T(a,b,c)= (2b+c, a-4b, 3a) (6+4)

4. a) Prove that, $\frac{d}{dx}[erf(\sqrt{x})] = \frac{e^{-x}}{\sqrt{\pi x}}$

b) Find out the value of the integral $\int_0^\infty \frac{\ln x}{(1+x^2)^2}dx$ (3+7)

5. a) Convert ordinary polynomial $16x^4 + 4x^3 - 8x^2 + 20x + 8$ into Hermite Polynomial

b) If $J_{n+1}(x) = \frac{2}{x}J_n(x) - J_0(x)$, find the value of n

c) The generating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x)t^n$ for the Legendre Polynomial $P_n(x)$ is $F(x,t) = (1 - 2xt + t^2)^{-1/2}$. Find the value of $P_3(-1)$

d) Show that $L_4^2(x) = 144 - 96x + 12x^2$ (3+2+3+2)

(P.T.O.)

PHS 101.2 CLASSICAL MECHANICS

Answer any TWO questions of the following: **2X10=2**

* Symbols have their usual meanings.

1. Explain Hamilton's principle. Find $[\dot{p}, H]$ and $[\dot{q}, H]$ and find the values of p and q for the Hamiltonian H= $(p^2+q^2)/2$, Also show that the energy is constant. What do you mean by stable, unstable and natural equilibrium? (3+5+2)

2. Prove that Possion's bracket remain invariant under canonical transformation. A particular mechanical system depending on two coordinates x and y has kinetic energy T = $\dot{x}^2y^2 + 2\dot{y}^2$ and potential energy V = x^2-y^2 , Write down the Lagrangian for the system and deduce its equation of motion with its solution. (4+6)

3. What kind of transformation is generated by the function F = - $\sum_i Q_i P_i$? Explain Exchange transformation and Identity transformation. For a dynamical system having q_i and p_i respectively the generalised coordinates and momenta and Hamiltonian H, derive the following relations $\dot{p}_i = -\frac{dH}{dq_i}$ and $\dot{q}_i = \frac{dH}{dp_i}$; (2+3+5)

4. (a) For a system consisting of a single particle show that the principle of least action becomes,

   $\Delta \int \sqrt{H-V}\, ds = 0,$ where ds = elementary path, H = Hamiltonian and V = Potential energy.

   (b) Determine the oscillations of a system with two degrees of freedom whose Lagrangian is, L= $0.5(\dot{x}^2+\dot{y}^2) - 0.5\, w_o^2(x^2+ y^2) + \alpha xy$ (5+5)

5. (a) A particle moves in a plane under the influence of a force, whose magnitude is

   F = $\frac{1}{r^2}\left(1-\frac{\dot{r}^2-2\ddot{r}r}{e^2}\right)$, where r is distance of the particle to the centre of force. Find the potential that will result in such a force, and from that the Lagrangian for the motion in a plane.

   (b) Find out the Lagrangian of a particle of charge q, mass m and linear momentum p, enters an electromagnetic field of vector potential V and scalar potential A and obtain the Hamiltonian of the particle. (5+5)

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