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PG CBCS

M.SC. Semester-IV Examination, 2021

(MATHEMATICS)

PAPER: MTM- 404B

(**NONLINEAR OPTIMIZATION**)

Full Marks: 40 **Time: 2 Hours**

Answer any <u>FOUR</u> questions from the following: **4×10=40**

1. (a) State and prove the Fritz- John stationary point necessary optimality theorem.

   (b) Solve the following problem by Beale's method

   Maximixe $z = 2x_1 + 3x_2 - x_1^2 - x_2^2$
   subject to $x_1 + x_2 \leq 2$
   $x_1, x_2 \geq 0$ 4+6

2. (a) Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$.θ is convex if and only if $\theta(x^2) - \theta(x^1) \leqq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$.

   (b) Define the following terms:

   (i) The (primal) quadratic minimization problem (QMP).
   (ii) The quadratic dual (maximization) problem (QDP).

   5+5

3. (a) How do you solve the following geometric programming problem?

   Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

   $$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}} \right)$$

   $c_j > 0, x_i > 0, a_{ij}$ are real numbers, $\forall i,\ j$.

   (b) Derive the Kuhn-Tucker conditions for the quadratic programming problem. 7+3

4. (a) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem:

   (i) Pareto optimal solution
   (ii) Weak Pareto optimal solution

   (b) Give the geometrical interpretations of differentiable convex function and concave function. 5+5

5. (a) State and prove Fritz-John saddle point sufficient optimality theorem. What are the basic differences between the necessary criteria and sufficient criteria of FJSP.

(b) Define the following:
   (i) Minimization problem
   (ii) Local minimization problem. 6+4

6. (a) Solve by using Wolfe's method the following quadratic programming problem

$$Max\ z = 2x_1 + 3x_2 - 2x_1^2$$
$$Sub.to\ x_1 + 4x_2 \leq 4$$
$$x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

(b) State and prove weak duality theorem in connection with duality in non-linear programming. 7+3

7. (a) What is the degree of difficulty in connection with Geometric programming.

(b) Define Nash strategy and Nash equilibrium outcome.

(c) What is stochastic programming problem?

(d) State Kuhn-Tucker stationary optimality theorem.

(e) State Karlin's constraint qualification. 2+2+2+2+2

8. (a) Find the Nash equilibrium solution(s) of the following matrix game(if exists)

$$\begin{bmatrix} (-2,-1) & (1,1) \\ (-1,2) & (-1,-2) \end{bmatrix}.$$

(b) Discuss chance constrained programming technique when only c_j are random variable. 4+6

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