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**PG CBCS**

**M.SC. Semester-IV Examination, 2021**

**(Mathematics)**

PAPER: MTM-403

(**MAGNETO HYDRO-DYNAMICSand STOCHASTIC PROCESS AND REGRESSION**)

**Full Marks: 40** **Time: 2 Hours**

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**Write the answers of each unit in separate sheets**

**PAPER: 403.1**

(**MAGNETO HYDRO-DYNAMICS)**

**Answer any TWO questions from the following**: **2×10=20**

1. (a) Explain, how to work Magneto-Hydrodynamics as a power generator.

(b) Define magnetic Reynolds number and explain its physical significance.

(c) For a conducting fluid in a magnetic field, show that the magnetic body force per unit volume, i.e. $\mu(\vec{\nabla} \times \vec{H}) \times \vec{H}$ is equivalent to a tension $\mu H^2$ per unit area along the lines of force, together with a hydrostatic pressure $\frac{1}{2}\mu H^2$, where symbols have their usual meaning. **2+2+6**

2. (a) A viscous, incompressible conducting fluid of uniform density are confined between a channel made by an infinitely conducting horizontal plate $z = -L$ (lower) and a horizontal infinitely long non-conducting plate $z = L$ (upper). Assume that a uniform magnetic field $H_0$ acts perpendicular to the plates. Both the plates are in rest. Find the velocity of the fluid and the magnetic field. Draw a sketch of the velocity profiles for various values of the Hartmann number.

(b) Find the rate of change of magnetic energy in magneto-hydrodynamics. **7+3**

3. Discuss the Ferraro's law of isorotaion. Show that the electrostatic potential over an isorotational surface is constant. **10**

4. (a). Derive the Maxwell's electromagnetic field equation when the medium is in motion.

(b). What are the boundary conditions for a fluid surface in presence of magnetic field? **6+4**

## PAPER: 403.2

**(STOCHASTIC PROCESS AND REGRESSION)**

**Answer any <u>TWO</u> questions from the following**: **2×10=20**

1. State and prove Chapman-Kolmogorov equation for a homogeneous Markov chain $\{X_n\}$. Suppose a two state homogeneous Markov chain has the following transition probability matrix:

   $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 \le a, b \le 1, |1-a-b| < 1.$

   Prove that (by using Chapman-Kolmogorov equation) the n-step transition probability matrix$P(n)$ is given

   $$P(n) = \begin{bmatrix} \frac{b+a(1-a-b)^n}{a+b} & \frac{a-a(1-a-b)^n}{a+b} \\ \frac{b-b(1-a-b)^n}{a+b} & \frac{a+b(1-a-b)^n}{a+b} \end{bmatrix}.$$ **5+5**

2. Show that $r_{12}, r_{13}$ and $r_{23}$ must satisfy the inequality $r_{12}^2 + r_{13}^2 + r_{23}^2 - 2r_{12}r_{13}r_{23} \le 1$. Prove that $r_{1.23\ldots p} = \left(1 - \frac{|R|}{R_{11}}\right)^{1/2}$where the symbols have their usual meanings. **2+8**

3. Write the postulates of Poisson process. Establish the Poisson law using the postulates of Poisson process. **2+8**

4. Write the transition probability matrix for the Gambler's ruin problem. Consider a Markov chain on the state space $\{1, 2, 3, 4, 5\}$with transition probability matrix $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{1}{4} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{2}{5} & \frac{1}{5} & 0 & \frac{2}{5} & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$.Identify the states as recurrent and transient. **4+6**

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