# PG CBCS <br> M.SC. Semester-III Examination, 2021 <br> DEPARTMENT OF MATHEMATICS <br> PAPER: C-MTM-304 <br> (DISCRETE MATHEMATICS) 

## Full Marks: 50

Time: 2 Hours
Answer any FOUR questions from the following: $10 \times 4=40$

1. a) Define regular graph. Prove that the sum of the degrees of all vertices of a graph is an even integer.
b) Prove that the complement law $a+a^{\prime}=1$ and $a \cdot a^{\prime}=0$ in a Boolean algebra.
c) State and prove De-Morgan's law in a Boolean algebra. $4+2+4$
2. State the principle of inclusion-exclusion. Use the principle of inclusionexclusion, find the number of primes less than 100.
3. a) Explain binary tree. Find the number of pendant vertices in a binary tree with $n$ vertices.
b) Define poset and show that the set $Z^{+}$of all positive integers under divisibility relation forms a poset.

5+5
4. a) A function $f$ is defined by $f(x, y, z)=y z+y^{\prime} z^{\prime}$. Find its conjunctive normal form (CNF) and disjunctive normal form (DNF).
b) Define centre of a graph. Show that every tree has either one or two centre.
5. a) Show that if a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices.
b) Prove that by mathematical induction $n<2^{n}$, for all natural number $n$.
c) What is proposition?
6. a) Prove that a tree T with n vertices has $\mathrm{n}-1$ edges.
b) Write down the differences between conjunctive normal form and disjunctive normal form.
c) Prove $\sim(p \wedge q) \vee q$ is a tautology.
7. a) Prove that the power set of a set $X$ together with union, intersection and complement form a Boolean algebra.
b) Define generating function and find a closed form for the generating function of the sequence $0,1,2,3, \ldots \ldots$. $6+4$
8. a) Convert 257 into corresponding binary number.
b) Write a short note on Full Adder.
c) Draw a Full Adder using Half Adder.
d) Define connected graph.
$2+4+2+2$
[Internal Assessment-10 Marks]

