Total pages: 2

PG CBCS

M.SC. Semester-III Examination, 2021

DEPARTMENT OF MATHEMATICS

PAPER: MTM 302

(INTEGRAL TRANSFORMS AND INTEGRAL EQUATIONS)

Full Marks: 50 **Time: 2 Hours**

Answer any <u>FOUR</u> questions of the following: **10X4=40**

1. Answer all questions. 5x2=10
   i. If $*$ is the convolution operator concerning on Laplace transform, then show the operator $*$ is commutative.
   ii. Define Fourier transform and state the conditions of existence of the transform.
   iii. Define eigen value and eigen vector in terms of an integral equation.
   iv. Verify the initial value theorem in connection with Laplace transform for the function $(4 + t)^2$.
   v. State Bromwich's integral formula concerning on inverse Laplace transform.

2. a) State and prove convolution theorem on Laplace transform.
   b) Solve the integral equation

   $$\varphi(x) = \int_0^x \frac{1}{(x-t)^\alpha} y(t)dt, 0 < \alpha < 1.$$ 5+5

3. a) Use the Laplace transform technique to solve the differential equation:

   $$\frac{d^2x}{d^2t} + 3\frac{dx}{dt} + 2x = 2t - e^{-t} \text{ which satisfies } x(0) = \frac{1}{2}, \frac{dx}{dt} = 0.$$

   b) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform. 5+5

4. a) Solve the following boundary value problem in the half plan $y > 0$, described by PDE: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $-\infty < x < \infty, y > 0$ with boundary conditions $u(x, 0) = f(x), -\infty < x < \infty$. u is bounded as $y \to \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \to \infty$.

   b) Find the value of $\sin(t) * t^2$ where $*$ denotes the convolution operator on Laplace transform. 7+3

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5. a) Solve the integral equation

$$y(x) = f(x) + \lambda \int_{-1}^{1} (xt + x^2t^2) y(t) dt.$$

b) State initial value theorem in respect of Laplace transform. Evaluate

$$L\left\{\int_0^t \frac{\sin u}{u} du\right\}$$

by the help of initial value theorem. 5+5

6. a) Verify the final value theorem in connection with Laplace transform for the function $t^3 e^{-t}$

b). Reduce the boundary value problem $\frac{d^2y}{d^2x} + \lambda xy = 1, 0 \le x \le l$ with boundary conditions $y(0) = 0, y(l) = 1$ to an integral equation and find its kernel.3+7

7. a) Find the resolvent kernel of the following integral equation and then solve it:

$$\varphi(t) = t^2 + \int_0^x \sin(t - y)\varphi(t) dt$$

b) Show that the Fourier transform of $\frac{a}{x^2+a^2}$, $(a > 0)$, is $\sqrt{\frac{2}{x}} e^{-a|k|}$ where k is the Fourier transform parameter. 6+4

8. a) Solve the integral equation

$$y(x) = x + \lambda \int_{-\pi}^{\pi} [x\cos(t) + t^2 \sin(x) + \cos(x) \sin(t)] y(t) dt$$

b) Evaluate $\left\{\int_0^t J_0(s) J_1(t-s) ds\right\}$ 7+3

[**Internal Assessment-10 Marks**]