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PG CBCS

M.SC. Semester-III Examination, 2021

DEPARTMENT OF MATHEMATICS

PAPER: MTM-301

(PARTIAL DIFFERENTIAL EQUATIONS AND GENERALIZED FUNCTIONS)

Full Marks: 50 **Time: 2 Hours**

Answer any <u>FOUR</u> questions from the following: **10×4=40**

1. a) Solve the following: $\left(D^2 + 5DD' + D'^2\right)z = 0$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$
   b) Find the derivative of the Heaviside unit step function.
   c) Find the solution of $z^2 = pqxy$. 4+3+3
2. a) Show that the following equation is hyperbolic:
   $$U_{xx} + 6U_{xy} - 16U_{yy} = 0.$$
   b) Find the canonical form of the equation.
   c) Find the general solution $U(x, y)$.
   d) Find a solution $U(x, y)$ that satisfies $U(-x, 2x) = x$ and $U(x, 0) = \sin 2x$.
   1+4+2+3
3. a) Show that the Green's function for the Laplace equation is symmetric.
   b) Establish the Poisson's formula for the solution of a Dirichlet problem for the Laplace equation in a disk of radius a. 4+6
4. a) Let $Lu = a(x)\frac{\partial^2 u}{\partial x^2} + b(x)\frac{\partial u}{\partial x} + c(x)u$. Find the adjoint operator of L.
   b) Prove that a Laplace operator is a self-adjoint operator. 6+4
5. a) Prove that $\delta(at) = \frac{1}{a}\delta(t)$. Symbols have their usual meaning.
   b) Solve the following problem:
   $$u_t = u_{xx} - u, 0 < x < 1, t > 0$$
   $$u(0,t) = u_x(1,t) = 0, t \geq 0$$
   $$u(x,0) = x(2-x), 0 \leq x \leq 1.$$ 3+7

[P. T. O]

6. a) Find the general solution of the following PDE:

$$(D-2D'+5)(D^2+D'+3)z=\sin(2x+3y). \text{ where } D\equiv\frac{\partial}{\partial x} \text{ and } D'\equiv\frac{\partial}{\partial y}$$

b) Solve, $p^2+q^2={n_0}^2$ where n_0 is constant with initial condition $\Gamma: U(x,2x)=1$. 5+5

7. Obtain the solution of the interior Dirichlet problem for the Poisson's equation in a sphere using the Green's function method. Hence derive the Poisson integral formula. 6+4

8. a) If $\beta_r D'+\gamma_r$, $(\beta_r>0)$ be a factor of $F(D',D)$ and $\varphi_r(\zeta)$ is an arbitrary function of the variable ζ, then prove that a solution of the equation $F(D',D)z=0$ is given by $U_r=e^{-\frac{\gamma_{r,}y}{\beta_r}}\varphi_r(\beta_r x)$, where β_r,γ_r are constants and $D\equiv\frac{\partial}{\partial x}$, $D'\equiv\frac{\partial}{\partial y}$.

b) Solve, $(\mathrm{D}+\mathrm{D}')^2 z=x^2+xy+y^2$. 5+5

[Internal Assessment- 10 Marks]