**PG CBCS**

**M.SC. Semester-II Examination, 2021**

**(MATHEMATICS)**

PAPER: MTM-203

(**ABSTRACT ALGEBRA AND LINEAR ALGEBRA**)

**Full Marks: 40** **Time: 2 Hours**

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**Write the answers of each unit in separate sheet**

**UNIT-203.1**

**(ABSTRACT ALGEBRA)**

**Answer any <u>Two</u> questions from the following**: **2×10=20**

1. (a) ($i$) With proper justification, give an example of an infinite non-commutative solvable group.
   ($ii$) Find the class equation of the group $S_4$.
   (b)Find the prime ideals and the maximal ideals in the ring $(Z_8, +, .)$.
   (2+3)+5
2. (a) Show that any epimorphism from the group $(Z, +)$ onto itself is an isomorphism.
   Is it true that any group of order 175 has a normal subgroup of order 25?
   (b) Show that in the ring $(\mathbb{Z} \times \mathbb{Z}, +, \ .)$, the set $I = \{(x, 0): x \in \mathbb{Z}\}$ is a prime ideal but not a maximal ideal. (2+3)+5

3. (a)Classify all groups of order 49 up to isomorphism.
   (b) Prove that every finite integral domain is a field. 6+4

4. (a) Define algebraic numbers for a field extension, and prove that sum of two algebraic numbers is also algebraic.
   (b) Is there exists an infinite group G, such that each element of G is of finite order? **7+3**

**UNIT-203.2**

**(LINEAR ALGEBRA)**

**Answer any <u>Two</u> questions from the following**: **2×10=20**

5. (a)What is the quotient space in linear algebra?
   (b) What is the linear functional on a vector space with examples.
   (c) Let T be linear operator on finite dimensional vector space V. When u say that T is diagonalizable?
   (d) Define Jordan block with example.
   (e) Define characteristic value and vector of a linear operator on a vector space. 2+2+2+2+2
6. (a)Find all possible Jordan canonical forms for a linear operator T: V to V (vector space) whose characteristicpolynomial is $(t-2)^3(t-5)^5$. In each case, find the minimal polynomial m(t).

(b) State and prove first Isomorphism theorem on linear algebra.

(c) Let $T: M_{2\times3}(F) \to M_{2\times2}(F)$be defined by

$$T\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \begin{pmatrix} 2a_{11} - a_{12} & a_{13} + 2a_{12} \\ 0 & 0 \end{pmatrix}$$

Find Null space of $T$ and Range space of $T$. Determine whether $T$is one-to-one or onto. 3+4+3

7. (a) Suppose V has finite dimension and dim V= dim U. Suppose $F: V \to V$ is linear. Then F is an isomorphism iff F is non-singular.

   (b) Let A be a real square matrix. Is A similar to a Jordan matrix? If not, give a counter example.

   C. Describe all canonical nilpotent matrices of order 3. 5+3+2

8. (a) Extend {(2, 3, -1), (1, -2,-4)} to an orthogonal basis of the Euclidean space $\mathbb{R}^3$ with standard inner product and then find the associated orthogonal basis.

   (b) Consider the basis $\{v_1 = (2,1), v_2 = (3,1)\}$ of $\mathbb{R}^2$. Find the dual basis $\{\varphi_1, \varphi_2\}$. (3+2)+5

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