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PG CBCS

M.SC.Semester-II Examination, 2021

(Mathematics)

PAPER: MTM-202

(NUMERICAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

Answer any FOUR questions from the following: **4×10=40**

1. (a) Find whether the following function is spline or not?

$$f(x) = \begin{cases} -x^2 - 2x^3, & x \in [-1, 0] \\ -x^2 + 2x^3, & x \in [0, 1] \end{cases}$$

(b) Develop the cubic spline of the following information

x:	1	2	3	4
f(x):	1	5	11	8

where $y''(1) = 0 = y''(4)$. Hence compute $y(1.5)$. 3+7

2. (a) Discuss the Newton-Raphson method for a pair of non-linear equations with stated convergence conditions.

(b) Approximate the function $\sin x, -1 \leq x \leq 1$ using Chebyshev polynomials. 5+5

3. (a) Use fourth order Runge-Kutta method to solve the second order initial value problem $2y''(x) - 6y'(x) + 2y(x) = 4e^x$ with $y(0) = 1$ and $y'(0) = 1$ at $x = 0.2, 0.4$.

(b) Discuss Milne's predictor-corrector formula to find the solution of $y' = f(xy), y(x_0) = y_0$. 5+5

4. (a) Use Jacobi's method to determine all eigenvalues and the eigenvectors of the real symmetric matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$.

(b) Analyze the stability of Runge-Kutta method for initial value ODE.

7+3

[P.T.O]

5. (a) Derive the Gauss-Chebyshev quadrature formula. Using six points Gauss-Chebyshev quadrature formula evaluate $\int_0^2 \frac{x}{1+x^3}dx$.

(b) Explain Monte Carlo method to integrate $\int_a^b f(x)dx$. 6+4

6. (a) Explain the ill-conditioned and well- conditioned system. The coefficient matrices of two system of equations are $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} and B = \begin{pmatrix} 1 & 1 \\ 1 & 1.0001 \end{pmatrix}$. Find the condition numbers of two systems and indicate which system is stable.

(b) Derive the Gauss-Legendre quadrature formula to integrate $\int_{-1}^{1} \psi(x)f(x)dx$. 5+5

7. (a) Solve the system of equations

$$2x + 4y - 2z = 14$$
$$x + 3y - 4z = 16$$
$$-x + 2y + 3z = 1$$

using LU-decomposition method.

(b) Express the polynomial $x^4 + 2x^3 - x^2 + 5x - 9$ in terms of Chebyshev polynomials. 8+2

8. (a) Describe an implicit method to solve a parabolic PDE.

(b) Solve the boundary value problem $y'' + xy' + 4 = 0, y(0) = 0 \text{ and} y(1) = 0$ with step length $h = 0.25$. 5+5

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