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PG CBCS

M.SC. Semester-I Examination, 2021

MATHEMATICS

PAPER: MTM-105

(CLASSICAL MECHANICS AND NONLINEAR DYNAMICS)

Full Marks: 50 **Time: 2 Hours**

Answer any FOUR questions from the following: **10×4=40**

1. Prove that:

$$J = \int_{x_0}^{x_1} F(y_1, y_2, \dots, y_n, y_1', y_2', \dots, y_n', x)dx$$

will be stationary if $y_1, y_2, \dots, y_n$ are obtained by solving the following equations:

$$\frac{d}{dx}\left(\frac{\partial F}{\partial y_j'}\right) - \frac{\partial F}{\partial y_j} = 0, \qquad j = 1,2, \dots n$$

where $y_j' = \frac{\partial y_j}{\partial x}$. 10

2. A body moves about a point Q under no forces. The principal moments of inertia at O being 3A, 5A and 6A. Initially, the angular velocity has components $w_1 = n, w_2 = 0, w_3 = n$ about the corresponding principal axes. Show that at any time t,

$$w_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$$

and that the body ultimately rotates about the mean axis. 10

3. State Hamilton's principle and derive it from D'Alembert's principle. 2+8

4. a) What is the effect of Coriolis force on a particle falling freely under the action of gravity.

b) Find the Lagrange's equation of motion for a pendulum of length l in spherical polar coordinates. 3+7

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5. A body of mass m_1 is thrown up an inclined plane which is moving horizontally with a constant velocity V. Use Lagrangian equation to find the locus of the position of the body at any time t, after the motion sets in. 10

6. a) If the equations of transformation do not depend explicitly on time and the potential energy is velocity independent, then prove that H is the total energy of the system.

   b) In special theory of relativity, show that

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$ 4+6

7. a) If $[X, Y]$ denotes the Poisson bracket, then prove the following results:

   (i) $[X+Y, Z] = [X, Z] + [Y, Z]$

   (ii) If $q = \sqrt{2P}\sin Q,\ \ p = \sqrt{2P}\cos Q$, then prove that $[Q, P] = 1$.

   b) Show that $E = mc^2$, in relativistic mechanics. 6+4

8. Show that the transformation

   $Q = \log(1 + \sqrt{q}\cos p)$, $P = 2\sqrt{q}(1 + \sqrt{q}\cos p)\sin p$ is canonical. Find the generating function $G(q, Q)$.

   Hence show that the generating function of this transformation can be put in the form $F = -(e^Q - 1)^2 \tan p$. 10

[Internal Assessment- 10 Marks]