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PG CBCS

M.SC. Semester-I Examination, 2021

MATHEMATICS

PAPER: MTM-103

(ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTIONS)

Full Marks: 50 **Time: 2 Hours**

Answer any FOUR questions from the following: **10×4=40**

1. a) Find the general solution of the homogeneous equation $\frac{d\vec{x}}{dt} = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} \vec{x}$ where $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$.

   b) Prove that $\frac{d}{dz}[z^{-n}J_n(z)] = -z^{-n}J_{n+1}(z)$, where $J_n(z)$ is the Bessel's function. 6+4

2. a) Find the power series solution near the origin of the confluent hypergeometric equation.

   b) Prove that $(\alpha - \beta)F(\alpha, \beta, \gamma, z) = \alpha F(\alpha + 1, \beta, \gamma, z) - \beta F(\alpha, \beta + 1, \gamma, z)$ where $F(\alpha, \beta, \gamma, z)$ is the hypergeometric function. 7+3

3. a) Find the series solution near z=0 of $(z + z^2 + z^3)w''(z) + 3z^2w'(z) - 2w(z) = 0$.

   b) Deduce the Rodrigue's formula for Legendre's polynomial. 6+4

4. a) Using Green's function, solve the boundary value problem $y'' + \pi^2 y = cos\pi x, y(0) = y(1), y'(0) = y'(1)$.

   b) Define fundamental matrix. 8+2

5. a) If α and β are the roots of the equation $J_n(z) = 0$ then show that
   $\int_0^1 J_n(\alpha z)J_n(\beta z)dz = \begin{cases} 0, & if \alpha \neq \beta \\ \frac{1}{2}[J_n'(\beta)]^2, & if\ \alpha = \beta \end{cases}$.

   b) Prove that $\int_{-1}^{1} P_m(z)P_n(z)dz = \frac{2}{2n+1}\delta_{mn}$, where $\delta_{mn}, P_n(z)$ are the Kronecker delta and Legendre's polynomial respectively. 6+4

[P. T. O]

6. a) If $f(z)$ is continuous and has continuous derivative on [-1, 1] then prove that $f(z)$ has unique Legendre series expansion given by $f(z) = \sum_{n=0}^{\infty} c_n p_n(z)$ where $p_n(z)$ is the Legendre's polynomial and $c_n = \frac{2n+1}{2}\int_{-1}^{1} f(z)p_n(z)dz\,, n = 0, 1, 2, \ldots$
   b) Deduce confluent hypergeometric differential equation from hypergeometric differential equation.
   c) Find the singularities of the confluent hypergeometric equation. 6+2+2
7. a) Find the characteristics values and characteristic functions of Strum-Liouville problem $(x^3y')' + \lambda xy = 0; y(0) = 0, y(e) = 0$.
   b) Consider the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0; 0 \le x \le \pi, y(0) = 0, y(\pi) = 0$. Find the values of λ for which the boundary value problem is solvable. 7+3
8. a) If the solution $\varphi_1, \varphi_2, \ldots, \varphi_n$ of the linear homogeneous vector differential equation $\frac{dx}{dt} = A(t)X(t)$ be a fundamental solution of above and $\varphi(t)$ be a arbitrary solution of above. Then prove that $\varphi(t)$ can be expressed as alinear combination of $\varphi_1, \varphi_2, \ldots, \varphi_n$ on [a, b].
   b) Prove that $F(\alpha, \beta, \gamma, z) = (1-z)^{-\alpha} F\left(\alpha, \gamma - \beta, \gamma, \frac{z}{z-1}\right)$.

[Internal Assessment- 10 Marks]