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PG CBCS

M.SC. Semester-I Examination, 2021

MATHEMATICS

PAPER: MTM 102

(COMPLEX ANALYSIS)

Full Marks: 50 **Time: 2 Hours**

Answer any <u>FOUR</u> questions of the following: 10X4=40

1. a) State and prove Rouche's theorem.
   b) (i) State Cauchy Integral formula.
   (ii) Using Cauchy Integral formula evaluate $\int_C \frac{z\,dz}{z^2-1}$, where $C: |z| = 2.$
   (2+3)+(2+3)

2. a) Using the calculus of residues evaluate $\int_0^\infty \frac{dx}{(x^2+4)^3}$.
   b) Let a function is defined by $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}, z \neq 0 \\ 0, z = 0 \end{cases}$. Show that the C-R equations are satisfied at (0,0) although f(z) is not differentiable at (0,0).
   6+4

3. a) Find the bilinear transformation that maps the upper half plane *Im(z)>0* onto the open disc $|w|<1$ and the boundary *Im(z)=0* onto the boundary $|w|=1$.
   b) Find the Mobious transformation that maps $1, 0, -1$ to the respective points $i, \infty, 1$
   4+6

4. a) Evaluate $\int_C \frac{\cosh(z)}{z(1+z^2)}dz$ where C is the circle $|z| = 2$, described in the positive sense.
   b) Using the single residue of $\frac{1}{z^2} f(\frac{1}{z})$ at z=0, evaluate the integral of $\frac{z^5}{1-z^3}$ around the circle |z|=2 in the positive sense.
   c) Find the value of $\int_C Log(z+4)dz$ when the contour C is ellipse $9x^2+4y^2=36$, in either direction.
   4+3+3

5. a) Define analytic function in a domain.
   b) Show that a real valued function of a complex variable either has derivative zero or the derivative does not exist.

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c) Show that the function $f(z) = |z|$ is not differentiable at $z = 0$.

(d) Is the function $u = 2xy + 3xy^2 - 2y^3$ harmonic? Justify your answer.

2+3+3+2

6. a) State Uniqueness theorem for power series.

b) Using the method of residue, evaluate: $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2+a^2)(x^2+b^2)}$.

c) Evaluate $\int_\gamma x\, dz$, where γ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

d) Define winding number of a curve. 2+3+3+2

7. a) Examine whether the following statement is true or false with proper justification: $\sin(z)$ is bounded in complex plane.

b) Prove that every zero of an analytic function $f(\not\equiv 0)$.

c) Suppose that D is a domain and $f \in \mathcal{H}(D)$ vanishes throughout any neighborhood of a point in D. Show that $f(z) \equiv 0$ in D. $3 + 4 + 3$

8. Examine whether the following statements are true or false with proper justification:

(i) An entire function $f(z)$ having $z = \infty$ as a removal singularity is constant.

(ii) The function $f(z) = \frac{z^4+1}{(\alpha z - |z|^2)(\beta z - |z|^2)}, \alpha, \beta \in \mathbb{C} \setminus \{0\}$ $(\alpha \neq \beta)$ has two simple poles at $z = \overline{\alpha}, \overline{\beta}$ and a double pole at $z = 0$.

(iii) There exists an analytic function in the unit disk Δ such that

$$f\left(\frac{1}{2n}\right) = f\left(\frac{1}{2n+1}\right) = \frac{1}{n} \text{ for } n \geq 2.$$

(iv) The zeros of $\sin\left(\frac{1}{z}\right)$ are $z = \frac{1}{n\pi}$ $(n \in \mathbb{Z})$ and each zero is isolated.

$2 + 3 + 3 + 2$

[Internal Assessment: 10 Marks]