
(c) Determine the following polynomials $u, v, w$ in $p(t)$ are linearly independent or not where $u=t^{3}-4 t^{2}+3 t+3, v=t^{3}+2 t^{2}+4 t-1, w=2 t^{3}-t^{2}-3 t+5 . \quad 8+8+4$
2. (a) Solve using Laplace transform methode $y^{\prime \prime}+2 y^{\prime}+5 y=0$ where $y=2, y^{\prime}=-4$ at $x=0$.
(ii) Express $f(t)=\left\{\begin{array}{cc}\sin t, & 0<t<\pi \\ t, & x>\pi\end{array}\right.$ In terms of unit step function and obtain Laplace transform.
(iii) Find the porjection of $(1,1,2,4)$ along $(1,1,1,1)$ in $R^{4}$.
(iv) Find the basis and dimension of the subspace $W$ of $V=M_{2,2}$ spanned by

$$
A=\left(\begin{array}{cc}
1 & -5 \\
-4 & 2
\end{array}\right) ; B=\left(\begin{array}{cc}
1 & 1 \\
-1 & 5
\end{array}\right) ; C=\left(\begin{array}{cc}
2 & -4 \\
-5 & 7
\end{array}\right) ; D=\left(\begin{array}{cc}
1 & -7 \\
-5 & 1
\end{array}\right) \quad 6+8+3+3
$$

3. (a) Let $F: R^{4} \rightarrow R^{3}$, be a linear mapping defined by
$F(x, y, z, t)=(x-y+z+t, 2 x-2 y+3 z+4 t, 3 x-3 y+4 z+5 t)$. Find the basis and dimension of Image of F as well as Kernel of F .
(b) Using Gram-Schmidt process find the orthonormal basis set of $\left\{1, t, t^{2}, t^{3}\right\}$. Given $\langle f, g\rangle=\int_{-1}^{1} f(t) g(t) d t$.
(c) Find the Laplace transform of $x J_{1}(x)$. Where $J$ represents the Bessel function.

$$
8+8+4=20
$$

4. (a) Derive the transformation law for Christoffel symbols of the first and the second kind.
(b) Show th at the matrix $\left[g^{i j}\right]$ is the inverse of the matrix $\left[g_{i j}\right]$. Hence calculate the contravariant components $g^{i j}$ of the metric tensor in cylindrical polar coordinate.
(c) Using tensor method prove the vector identity $\nabla\left(\frac{A^{2}}{2}\right)=\vec{A} \times(\nabla \times \vec{A})+(\vec{A} \cdot \nabla \vec{A})$.
(d) Find out $[22,1]$ in spherical polar coordinate.

## Group - B

PRACTICAL (Marks : 20)

Answer any one from the following questions :

1. Explain the procedure in details for any one of the following :
(i) Determination of the principal axes of moment of inertia through diagonalization.
(ii) To study vector space of wave functions in Qunatum Mechanics : Position and momentum differential operators and their commutator, wave functions for stationary states as eigen functions of Hermitian differential operator.
(iii) Study of Lagrangian formulation in Classical Mechanics with constraints.
(iv) Study of geodesics in Euclidean and other spaces (surface of a sphere, etc).

## APPLIED DYNAMICS <br> Group - A <br> THEORY (Marks : 40)

Answer any two from the following questions :

1. (a) Find all fixed points for $\dot{x}=x^{2}-1$, and classify their stability. Draw the phase portrait. $5+2$
(b) Consider the electrical circuit where $a$ resistor $R$ and capacitor $C$ are in series with battery of constant dc voltage $V_{0}$. Suppose the switch is closed at $t=0$, and there is no charge on the capacitor initially. Sketch the graph of $Q(t)$, the charge on the capacitor as a function as a function of time. Also draw the phase portrait. $4+3$
(c) Sketch the phase portrait corresponding to $\dot{x}=x-\cos x$ and determine the stability of all the fixed points.
2. (a) Discuss the logistic model of population growth model with carrying capacity $K$ with a proper phase portrait.
(b) Show that it is impossible for the Lorenz system to have either repelling fixed points or repelling closed orbits.
3. What is chaos? Define attractor and strange attractor with example and diagram. 20
4. Define self-similarity and fractals with proper example. Show that the Cantor set is uncountable. Discuss von Koch curve. Show that the von Koch curve has a self-similarity dimension of 1.2.6.

## Group - B

PRACTICAL (Marks : 20)
Answer any one from the following questions :

1. Describe how you can visulaize trajectories in Sinai Billiard using softwares like Maple/ Octave etc.
2. Explain how it is possible to visualize and recreate the fractal formations in nature (trees, coastlines, earthquakes) using applied dynamics based softwares.
3. Explain how you can determine the coupling coefficient of coupled oendulums using software based on applied dynamics problems.

# ATMOSPHERIC PHYSICS <br> Group - A <br> THEORY (Marks : 40) 

Answer any $\boldsymbol{t} \boldsymbol{w o}$ from the following questions :

1. (a) Describe the thermal structure of the Earth's atmosphere.
(b) What do you mean by greenhouse effect? What is effective temperature of Earth?
(c) Explain cyclones, anticyclones and thunderstorms?
2. (a) Describe the vectorial form of the momentum equation in rotating coordinate system.
(b) What do you mean by atmosphereic oscillations ? Explain it is terms of the basic equations.
(c) Differentiate between general circulation and mesoscale circulation.
3. (a) Describe the propagation of atmospheric gravity waves in a non-homogeneous medium.
(b) What is Rossby wave ? Describe its propagation in three dimensions and in shared flow.
4. (a) Derive Radar range equation.
(b) Write a short note on various types of atmospheric radars with their application to study atmospheric phenomena.

## Group - B

PRACTICAL (Marks: 20)

Describe the procedure of any one of the following on the basis of $\mathrm{C}++$ simulation

1. (a) To study atmospheric gravity waves (AGW).
(b) To study Rossby waves and mountain waves.
2. Offline and Online processing of RADAR data. 20
3. Offline and Online processing LIDAR data.
4. Handling of satellite data and plotting of at mospheric parameters using radio occultation technique.

## CLASSICAL DYNAMICS THEORY (Marks : 60)

Answer any three from the following questions : $3 \times 20$

1. (a) What do you meant by four vector?
(b) Explain the terms flow line and stream line.
(c) What are the advantages of Hamiltonian mechanics over Lagrangaina mechanics?
(d) What do you mean by Intertial and non-intertial frame?
(e) A mass attached to a long spring has time period T . When the spring is cut into two parts and same mass is attached to one part, what will be the time period ?
2. (a) Find the velocity at which the mass of a particle becomes double of its mass.
(b) What do you mean by space like and time like interval in Minkowski's space ?
(c) The length contraction of a rod is found to be half of its own length. What is the relative speed of the rod to the observer ?
(d) Define zeroth component of four force. Signify it.
(e) State the basic postulates of Einsten's special theory of Relativity. Show by means of Lorentz transformation equations that $x^{\prime 2}-c^{2} t^{\prime 2}=x^{2}-c^{2} t^{2}$ Symbol are meaning the usual means.
3. (a) A particle of mass m is projected with initial velocity u at an angle $\alpha$ with the horizontal. Use Lagrange's equations to describe the motion of projectile.
(b) Lagrangian of an oscillator $L\left(x, x^{\prime}\right)=0.5 x^{\prime 2}-0.5 \omega^{2} x^{2}-a x^{3}+\beta x x^{\prime 2} ;\left(x^{\prime}=d x / d t\right)$ where $\alpha, \beta$ and $\omega$ are constants. Find the corresponding Hamiltonian.
(c) A uniform bar of length $L$, and mass $m$ is supported at the ends by identical springs of elastic constant $k$. Mention is initiated by depressing one end by a small distance ' $a$ ' and releasing it from rest.


Figure-1.

Prove that
$x=0.5(2 b-a) \cos \omega_{1} t ; \omega_{1}=(2 k / m)^{1 / 2}$
$\theta=(a / 1) \cos \omega_{2} t ; \omega_{2}=(6 \mathrm{k} / \mathrm{m})^{1 / 2}$

And the motion is in normal mode state.
4. (a) Two undamped one dimensional oscillator of natural frequencies are coupled together. Find the frequency of modes. Find the ratio of the amplitudes in these modes. Also introduce normal co-ordinate and set up the corresponding Lagrange's equations. 10
(b) (i) Give the necessary theory Poiseuille's method to determine the coefficient of viscosity of a liquid. State clearly the assumptions made.
(ii) Three tubes of equal lengths but of different radii are connected in series. Use Poiseuille's formula (without any correction) to obtain an expressing for the volume
of liquid flowing through the tube per second when the pressure difference between the two ends of the series is P. How the expression has to be modified if the tubes are connected in parallel ?
5. (a) (i) Prove that the total energy of a particle of mass $m$ acted upon by a crystal force is given by
$E=v(r)\left[u^{2}+\left(\frac{d u}{d t}\right)^{2}\right]\left(\frac{h^{2}}{2 m}\right)$; where $v(r)$ is the potential energy, $h$ is angular momentum and $(r, \theta)$ is the polar co-ordinate of the moving particle with $u=1 / r$.
(i) Establish the equation of continuity for motion of an ideal fluid. What contestation principle does it imply?
(b) What is static and dynamic pressure in a moving fluid ?
6. (a) Show that energy moment tensor by Lorentz Transformation is invariant.
(b) Why moving clock appears to run slow ?
(c) Derive the relativistic variation of man-energy relation $T=\left(m-m_{0}\right) c^{2}$. Where $T$ is kinetic energy of particle with mass $m$ and rest mass $m_{0}$.
(d) Two electrorns moved towards each other, the speed of each being 0.9 c in Galiean frame of reference. What is their speed relative to each other?
$6+2+6+6$

