UG/3rd Sem/PHS(H)/T/19

2019

B.Sc.

3rd Semester Examination PHYSICS (Honours)

Paper - C 5-T

Full Marks: 40

Time: 2 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer *five* questions from Group - A, *four* from Group - B and *one* from Group - C.

Group - A

Answer any five questions of the following:

2×5=10

1. State the type (parabolic, elliptic or hyperbolic) of the following partial differential equation.

(i)
$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

[Turn Over]

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(ii)
$$9 \frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial y^2} = 3x + 4y + 1$$
 2

- 2 Show that complex Fourier coefficient of odd function is purely imaginary. 2
- 3. What is the nature of singularity of the following differential equation?

$$y'' - \frac{6}{x^2}y = 0$$

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 2

A Lagrangian $L(q, q, t) = \frac{1}{2}mq^2 - \frac{1}{2}k(q - vt)^2$ 2

Find the generalised momentum and Hamiltonian of the system.

S. Prove that
$$erf(x) + erf_c(x) = 1$$

6. Fourier expansion of f(x) in the interval 0 < x < l

is:
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$
 show that

$$\int_0^L \left[f(x)^2 \right] dx = \frac{1}{4} a_0^2 + \frac{1}{2} \sum_{n=1}^\infty a_n^2$$

- 7. If L is the Lagrangian of a system, then show that $L_1 = L \pm \frac{dF}{dt}$ where F is a function of the generalized coordinates, momenta and time; will also satisfy Lagranege's equations.
- 8. Potential energy of a particle are given by:

$$V = \frac{A}{\sqrt{(x^2 + y^2 + z^2)}} - Bz^2 \ln(x^2 + y^2).$$

Find its generalised momenta p_{x} and p_{z} .

Group - B

Answer any four questions of the following:

 $4 \times 5 = 20$

9. Prove that $\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+n)^{m+1}}$, when n

is a positive interfer and m > -1.

10. A differential equation is given by:

$$(1-x^2)y'' - 2xy' + ny = 0.$$

[Turn Over]

- (a) Find the singular points.
- (b) Check whether the singular points are essential or non-essential. Comment whether series solution of this equation is possible or not.

2+1

Prove
$$\int_{0}^{\infty} e^{-(x+a)^{2}} dx = \frac{\sqrt{\pi}}{2} \left[1 - \operatorname{erf}(a) \right]$$

12. Write the integral $\int_0^1 \frac{x^3}{\sqrt{1-x^2}} dx$ in the form of Beta function and hence evaluate it.

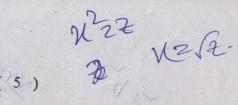
- 13. Calculate the Legendre transform of (i) $F(x) = x^2$ (ii) $F(x) = \ln x$. State the geometrically meaning of Legendre transform.
- 14. Derive Euler's equation of motion for couple oscillators.

Group - C

Answer any one question of the following:

 $10 \times 1 = 10$

15. (a) Solve the following boundary value problem by the method of separation of vaiaboles "



 $\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$; Given $u(0, y) = 8e^{-3y}$.

- (b) (i) Obtain the expression of kinetic energy of a particle in terms of generalized coordinates.
 - (ii) Show that in the absence of rheonomic constraints the Hamiltonian of a system is equal to the sum of kinetic and potential energies of the system.
 - (iii) Show that if the Lagrangian of a system does not depend on time explicitly then the Hamiltonian of this system remains conserved.
- 16. (a) Define error function erf(x). Find erf(0) and $erf(\infty)$. Draw the graph of error function.

1+2+1

(b) Find the solution of the following differential equation by the method of Frobenius:

y''-2xy'+2ny=0; where n is the non-negative integer.

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