বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2020

(Under CBCS Pattern)

Semester - V

Subject: MATHEMATICS

Paper: DSE1T

Full Marks : 60

Time : 3 Hours

Candiates are required to give their answer in their own words as far as practicable.

The figures in the margin indicate full marks.

LINEAR PROGRAMMING

Answer any ***three*** questions. 20×3=60

1. (a) What do you mean by feasible solution and basic feasible solution ? 4

(b) Use graphical method to solve :

Maximize $Z = x_1 + 0.5x_2$

Subject to the constraints $3x_1 + 2x_2 \ge 12$; $5x_1 = 10$; $x_1 + x_2 \ge 8$. 4

(c) A plastic manufacturer has 1200 boxes of transparent wrap in stock at one factory and another 1000 boxes at its second factory. The manufacturer has order for this product from three different retailers, in quantities of 1000, 700 and 1500 boxes respectively. The unit shipping cost from the factories to retailer are as follows :

Retailer → Factory ↓	I	II	III
A	14	13	11
B	13	13	12

Determine the minimum cost shipping schedule for satisfying all demands from current stock. Formulate it to LPP. 6

(b) Is the system of equation

$$x_1 + x_2 + x_3 = 4$$
$$2x_1 + 5x_2 - 2x_3 = 3$$
$$x_1 + 7x_2 - 7x_3 = 5$$

consistent ? Justify your answer. 6

2. (a) Define the term extreme point of a convex set. What is special feature of this point ? 4

(b) What do you mean by degeneracy of a simplex method ? When does it occur ? 4

(c) Show that basic feasible solutions are the extreme point of the convex set of feasible solutions of a LPP. 6

(d) State and prove the condition of unbounded solution of a maximization LPP, when we are going to solve it by simplex method. 6

3. (a) What do you mean by degeneracy in transportation problem. 4

(b) Describe the procedure to convert an assignment problem to a maximization problem. 4

(c) Find the optimal solution of the following LPP by solving its dual:

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to the constraints $x_1 + x_2 \geq 2$; $x_2 + x_3 \geq 5$; $x_1, x_2, x_3 \geq 0$. 6

(d) Show that the following problem has no feasible solution

Minimize $Z = x_1 - 3x_2$

Subject to the constraints $x_1 - x_2 \geq 3$; $-x_1 - x_2 \geq 2$; $x_1,\ x_2 \geq 0$. 6

4. (a) Show that in a balanced transportation problem, if the no. of source is m and destination is n, no. of basic variables will be $m+n-1$. 4

(b) Why do we study duality in LPP ? 4

(c) A company has three plants X, Y, Z and 3 warehouses A, B and C. The supplies are transported from the plants to warehouses which are located at varying distance from the plants. On account of the varying distance, the trnasportation costs from plants to warehouses vary from Rs. 12 to Rs. 24 per unit. The company wishes to minimize the transportation costs. The costs in Rs. from the plants to warehouses are as shown below :

	A	B	C	Supply
X	12	8	18	400
Y	20	10	16	350
Z	24	14	12	150
Demand	500	200	3000	

Determine the optimal shipping schedule. 6

(d) Solve the travelling salesman problem where the entries are given as distance. Find minimum distance.

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	¥

6

5. (a) Find the dual of the following LPP:

Maximize $Z = x_1 - x_2 + 3x_3 + 2x_4$

Subject to $x_1 + x_2 \geq -1$

$x_1 - 3x_2 - x_3 \leq 7$

$x_1 + x_3 - 3x_4 = -2$

$x_1, x_4 \geq 0$ and x_2, x_3 are unrestricted in sign. 4

(b) Why an assignment problem is not a LPP ? 4

(c) Find the optimal solution of the transportation problem using VAM method. 6

	D_1	D_2	D_3	D_4	D_5	
O_1	10	6	4	3	3	41
O_2	4	3	0	1	7	15
O_3	−1	4	−3	0	2	23
	19	10	4	8	15	

(d) Find the optimal assignments for the assignment problem with the following cost matrix.

	P	Q	R	S	T
A	85	75	65	125	75
B	90	78	66	132	78
C	75	66	57	114	69
D	80	72	60	120	72
E	76	64	56	112	68

Is the solution unique ? If not, identify an alternative solution. 6

6. (a) Define pure and mixed strategy of a Game. 4

(b) Define saddle point of a game and value of a game. 4

(c) Reduce the following pay-off matrix 2×2 by dominance property and hence solve the problem :

	Player B				
Player A	0	0	0	0	0
	4	2	0	2	1
	4	3	1	3	2
	4	3	4	–1	2

6

(d) Show that for a symmetric game the value of the game is zero. 6

POINT SET TOPOLOGY

Answer any ***three*** questions. 20×3=60

1. (a) Define the cardinal number of a set. 2

(b) Define accumulation point or limit point. 2

(c) Let $\mathbb{N}$ be the set of all positive integers and τ be a topology on $\mathbb{N}$ consisting of ϕ and all the sets of the form $E_n := \{n, n+1, n+2, ...\}$ where $n \in \mathbb{N}$. Then find the accumulation points of $A := \{5, 13, 28, 37\}$. Also determine those subsets of $\mathbb{N}$ whose derived set is $\mathbb{N}$. 2+3=5

(d) Let τ_1, τ_2 be two topologies on a non-empty set *X* with $\tau_1 \subseteq \tau_2$. Let $A \subseteq X$. Then prove that (i) every τ_2 limit point of *A* is a τ_1-limit point of *A*; (ii) give an example to show that a τ_1-limit point of *A* need not be a τ_2-limit point of *A*. 2+2=4

(e) Define finite intersection property. Then prove that a metric space (X, d) is compact if and only if for every collection of closed sets $\{F_\alpha : \alpha \in A\}$ in *X* possessing finite intersection property, the intersection $\bigcap_{\alpha \in \Lambda} F_\alpha \neq \phi$. 1+6=7

2. (a) State Hausdorff's maximal principle. 2

(b) Let X be a non-empty set. Define basis and sub-basis for a topology on *X* with examples. 2+2=4

(c) Let (X, τ) and (Y, U) be two topological spaces and $f : X \to Y$ be a mapping. Then prove that *f* is continuous if and only if *F* is closed subset in $Y \Rightarrow f^{-1}(F)$ is closed in *X*. 3

(d) Prove that a subset *S* of $\mathbb{R}$ is connected if and only if *S* is an interval. 5

(e) In a metric space (X, d), prove that a subset *A* of *X* is compact $\Rightarrow A$ is totally bounded. Is the converse implication true ? Give explanation in support of your answer. 3+3=6

3. (a) Prove that a countable union of countable sets is countable. 5

(b) Let X be a non-empty set and τ_1, τ_2 be two topologies on X with bases B_1, B_2, respectively. Show that if $\tau_1 \subseteq \tau_2$, then for $G_1 \in B_1$ and $x \in G_1$ there exists $G_2 \in B_2$ such that $x \in G_2 \subseteq G_1$. 2

(c) Let (X, τ) be a topological space and A be a non-empty subset of X. Prove that

(i) A^0 is the largest open set contained in A;

(ii) A is open if and only if $A = A^0$, where A^0 denotes the interior of A. 3+3

(d) Let Y be a subspace of X. Then show that Y is compact if and only if every covering of Y by sets open in X contains a finite subcollection covering Y. 5

(e) Give an example of a topology on $\mathbb{R}$, other than the indiscrete one, with respect to which $\mathbb{R}$ becomes compact. 2

4. (a) Let A and B two well-ordered sets. Prove that $A \times B$ is well-ordered in the Dictionary ordering. 5

(b) Let (X, τ) be a topological space such that A is a nowhere dense subset of X. Prove that $\overline{A}$ does not contain any non-emptry open set of X. 2

(c) Consider $\mathbb{R}$ with usual topology τ_u. Prove that $\mathbb{Q}$ is not a connected subspace of $(\mathbb{R}, \tau_u)$. 3

(d) Prove that the continuous image of a connected space is connected. 5

(e) Prove that a function f on a topological space to a product space is continuous if and only if the composition $\pi_\alpha \circ f$ is continuous for each projection $\pi_\alpha : \Pi_{\alpha \in \Lambda} X_\alpha \to X_\alpha$. 5

5. (a) Show that $\mathbb{Q}$ is countably infinite. 2

(b) In a topological space (X, τ), prove that a subset A of X is closed if and only if $bd(A) \subseteq A$ where $bd(A)$ denotes the boundary of A. 4

(c) Let (X, τ) be a topological space, (Y, τ_Y) be a subspace and $A \subseteq Y$. Then show that $\overline{A}_Y = \overline{A} \cap Y$ and $A^0 = \left(A^0\right)_Y \cap Y^0$. 2+2=4

(d) Let X_n be a metric space with metric d_n for $n \in \mathbb{N}$. Then show that

$$\rho(x, y) = \max\{d_1(x_1, y_1), d_2(x_2, y_2), ..., d_n(x_n, y_n)\}$$

where $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$, defines a metric on the product space $X_1 \times X_2 \times ... \times X_n$. 3

(e) Prove that $\mathbb{R}$ with usual topology has a countable base. 3

(f) If E is a connected subspace of a topological space X and F is a subset of X such that $E \subseteq F \subseteq \overline{E}$, prove that F is also a connected subspace of X. 4

6. (a) Prove that a finite product of countable sets is countable, using induction hypothesis. 5

(b) Show that the product topology on $\Pi_{\alpha\in\Lambda} X_\alpha$ is the coarsest topology relative to any topology on $\Pi_{\alpha\in\Lambda} X_\alpha$ where each projection $\pi_\alpha : \Pi_{\alpha\in\Lambda} X_\alpha \to X_\alpha$ is continuous. 4

(c) Prove that a topological space X is connected if and only if the only subsets which are both open and closed in X are ϕ and X only. 3

(d) Show that a topological space (X, τ) is disconnected if and only if there exists a continuous mapping from X onto the discrete two-point space $\{0, 1\}$. 4

(e) Define locally compact topological space with an example. 2

(f) State the Lebesgue number lemma. 2

THEORY OF EQUATIONS

Answer any ***three*** questions. 3×20=60

1. Use the method of synthetic division to find the quotient and remainder, when $x^5 - 4x^4 + 8x^2 - 1$ is divided by $(x-3)$. Find the condition so that $x^3 + 3px + q$ may have a factor of the form $(x-a)^2$. Express $x^4 + 5x^2 - 3x + 2$ as a polynomial in $(x+2)$. 20

2. Find the equation of fourth degree with rational coefficient, one root of which is $\sqrt{2} + \sqrt{3}i$. State Descartes' rule of sign. Use it to determine the nature of roots of the equation $x^n - 1 = 0$. Find the multiple root of the equation $x^4 + 2x^3 + 2x^2 + 2x + 1 = 0$. 20

3. Solve the equation $4x^3 + 16x^2 - 9x - 36 = 0$, when the sum of two roots is zero. Find the condition that the equation $x^3 + px^2 + qx + r = 0$ may have two equal roots but of opposite sign. If *m*, *n* are integers prime to each other then prove that 1 is the only common root of the equations $x^m - 1 = 0$ and $x^n - 1 = 0$. 20

4. Find the equation whose roots are the squares of the roots of the equation $x^3 + bx^2 + cx + d = 0$. Solve the cubic equation $x^3 - 18x - 35 = 0$ by Cardan's method. Find the relation between the coefficients of the equation $x^3 + ax^2 + bx + c = 0$, when the roots α, β are connected by the relation $1 + \alpha\beta = 0$. 20

5. Solve the equation $x^4 - x^3 + 2x^2 - x + 1 = 0$ which has four distinct roots of equal moduli. Show that the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 + (x-d)^3 = 0$ where a, b, c, d are positive and not all equal has only one real root. Remove the second term of the equation $x^4 + 4x^3 - 7x^2 - 22x + 24 = 0$ and hence solve the original equation. 20

6. Prove by Strum's theorem that the roots of the equation $x^4 - 4x^3 + 2x^2 + 4x + 1 = 0$ lie in the intervals (–1, 0) and (2, 3). Prove that the solution of any reciprocal equation depends on that of a reciprocal equation of first type and of even degree. If α, β, μ be the roots of the equation $x^3 + 5x^2 + 1 = 0$ find the value of $\sum \frac{1}{\alpha}$. 20