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UG/2nd Sem/Math/H/19

2019

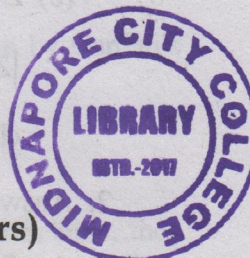
B.Sc.

2nd Semester Examination

MATHEMATICS (Honours)

Paper - GE2T

(Algebra)



Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Unit - I

(Classical Algebra)

[Marks - 22]

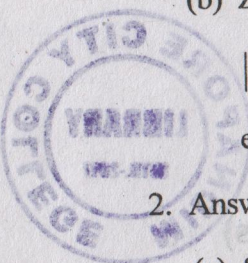
1. Answer any *one* question : 1×2

(a) Apply Descartes' rule of signs to find the number of imaginary roots of

$$3x^4 + 4x^2 - 3x - 12 = 0.$$

[Turn Over]

(2)



(b) Z is a variable complex number such that $|Z|=2$. Show that the point $Z + \frac{1}{Z}$ lies on an ellipse.

2. Answer any two questions: 2×5

(a) Show that the solutions of the equation

$$(1+x)^n - (1-x)^n = 0 \text{ are } x = i \tan \frac{r\pi}{n},$$

where $r = 0, 1, 2, \dots, n-1$, if n be odd.

$$= 0, 1, 2, \dots, \frac{n}{2}-1, \frac{n}{2}+1, \dots, n-1$$

if n be even.

(b) Prove that $\sum_{i=1}^8 a_i \geq 8 \left(\prod_{i=1}^8 a_i \right)^{\frac{1}{8}}$, where

$$a_i \geq 0 \quad \forall i = 1, 2, \dots, 8.$$

(c) Solve the equation $2x^4 + x^3 + 2x^2 + 3x + 18 = 0$, given that the product of two of the roots is equal to the product of the other two. 5

(3)

3. Answer any one question: 1×10

(a) (i) Solve the following equation by Cardan's method,

$$x^3 - 6x - 9 = 0.$$

(ii) If a, b, c be the sides of the triangle, then show that

$$\frac{1}{2} < \frac{bc+ca+ab}{a^2+b^2+c^2} \leq 1 \quad \text{5+5}$$

(b) (i) Find the relation among p, q, r, s so that the product of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is unity.

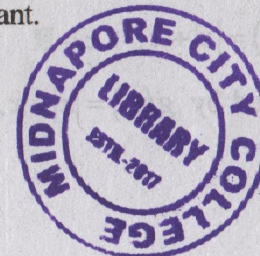
(ii) Find the products of all the values of

$$(1+i)^{\frac{4}{5}}.$$

(iii) Prove that the minimum value of

$$x^2 + y^2 + z^2 \text{ is } \left(\frac{C}{7} \right)^2 \text{ where } x, y, z \text{ are}$$

positive real numbers subject to the condition $2x+3y+6z=C$, C being a constant. 4+3+3



[Turn Over]

(4)

Unit - II

(Sets and Integers)

[Marks - 15]

4. Answer any five questions : 5×2

(a) Prove that $10^{n+1} + 10^n + 1$ is divisible by 3
 $\forall n \in \mathbb{N}$.

(b) State the second principle of mathematical induction.

(c) If A , B and C be three subsets of a universal set S prove that

$$A - (B \cup C) = (A - B) \cap (A - C).$$

(d) Define equivalence relation.

(e) Give an example to establish that the union of two transitive relations may not be a transitive relation.

(f) Let $A = \{x \in \mathbb{Z} : 0 \leq x \leq 10\}$ and

$B = \{x \in \mathbb{Z} : 5 \leq x \leq 15\}$ be two sets. Prove that the cardinality of two sets A and B are equal.

(g) Find $f \circ g$, if $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$f(x) = |x| + x$, $x \in \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ is

defined by $g(x) = |x| - x$, $x \in \mathbb{R}$.

(5)

(h) If a is prime to b , prove that a^2 is prime to b .

5. Answer any one question : 1×5

(a) Show that $\gcd(a, a+2) = 1$ or 2 for every integer. 5

(b) (i) State the fundamental theorem on arithmetic.

(ii) Prove that the cube of any integer is of the form $9K$ or $9K \pm 1$. 1+4

Unit - III

(System of Linear Equations)

[Marks - 9]

6. Answer any two questions : 2×2

(a) For what value of a the following system of equations have no solution

$$ax + 2y = 3$$

$$2x + ay = 5 - a$$

(b) Is any solution of $AX = B$ (where $B \neq \underline{0}$) linearly dependent? Justify? 1+1

[Turn Over]

(6)

- (c) Find the solution of the system of equations in rational numbers.

$$\begin{aligned}x + 4y + z &= 0 \\4x + y - z &= 0\end{aligned}$$

7. Answer any *one* question : 1×5

- (a) Investigate for what values of λ and μ the following equations

$$\begin{aligned}2x + 3y + 4z &= 10 \\y + 2z &= 4 \\x + 2y + \lambda z &= \mu\end{aligned}$$

have (i) no solution, (ii) a unique solution and (iii) an infinite number of solutions. 2+2+1

- (b) (i) Find a row echelon matrix which is row equivalent to

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

- (ii) For what value of K the planes $x - 4y + 5z = K$, $x - y + 2z = 3$ and $2x + y + z = 0$ intersect in a line ? 3+2

(7)

Unit - IV

(Linear Transformation and Eigen Value)

[Marks - 14]

8. Answer any *two* questions : 2×2

- (a) Find the rank of the matrix

$$\begin{bmatrix} 4 & 6 & 8 \\ 5 & 5 & 6 \\ 2 & 1 & 1 \end{bmatrix}$$

- (b) If λ be an eigen value of an $n \times n$ matrix A , then show that $\frac{1}{\lambda}$ ($i\lambda \neq 0$) is an eigen value of A^{-1} .

- (c) Let $T: R^3 \rightarrow R^3$ defined by

$$T(x, y, z) = (yz, zx, xy), (x, y, z) \in R^3$$

Examine whether T is linear or not.

9. Answer any *one* question : 1×10

- (a) (i) Verify Cayley-Hamilton theorem for the matrix A . Express A^{-1} as a polynomial in A and then compute A^{-1} .

[Turn Over]

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{pmatrix} \quad 4+1+1$$

(ii) Find the eigen values of the matrix $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$
and find the eigen vectors for one eigen
value of A . 4

(b) (i) Let $V = \{(x, y, z) : x, y, z \in \mathbb{R}\}$, where \mathbb{R}
is a field of real numbers.

Show that

$W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ is a
subspace of V over R . Find the dimension
of W . 5

(ii) Determine the linear mapping $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$
which maps the basis vectors $(1, 0, 0)$,
 $(0, 1, 0)$, $(0, 0, 1)$ of \mathbb{R}^3 to the vectors
 $(1, 1)$, $(2, 3)$, $(3, 2)$ respectively. 5