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2019

B.Sc.

1st Semester Examination

MATHEMATICS (Honours)

Paper - GE 1-T

(Calculus Geometry and Differential Equation)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Unit - I

1. Answer any *three* questions : 3×2=6

(a) If $y = x^{n-1}\log x$, prove that $y_n = \frac{(n-1)!}{x}$.

(b) Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

[Turn Over]

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(c) Find the envelope of

$$y = mx + a\sqrt{1+m^2},\ \text{parameter } m.$$

(d) State Leibnitz's rule for successive differentiation. Find the n-th order derivative of $\cos(ax+b)$, where a, b are constants. 1+1

(e) Find the points of inflexion of the curve $y=(x+1)\tan^{-1}x$. 2

2. Answer any *one* question : 1×10=10

(a) (i) Trace the curve

$$(x^2+y^2)x-a(x^2-y^2)=0,(a>0)$$ 5

(ii) If $y^{\frac{1}{m}}+y^{-\frac{1}{m}}=2x$, then prove that

$$(x^2-1)y_{n+2}+(2n-1)xy_{n+1}+(n^2-m^2)y_n=0$$ 5

(b) (i) Find the asymptotes of the curve

$$y^3+x^2y+2xy^2-y+1=0$$ 5

(ii) Find the ranges of values of x in which the curve $y=3x^5-40x^3+3x-20$ is concave upwards or downwards. Also find the points of inflexion. 5

Unit - II

3. Answer any *two* of the following : 2×2=4

(a) Find the perimeter of the astroid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

(b) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

(c) Calculate the area bounded by the curves $y = x^2$ and $x = y^2$.

4. Answer any *two* questions : 2×5=10

(a) Show that the volume of the solid generated by the revolution of the curve $y(x^2 + a^2) = a^3$ about the asymptote of the curve is $2\pi^2a^3$.

[Turn Over]

(b) Find the reduction formula of $\int \sin^p x \cos^q x dx$, where q is positive integer and p is negative integers Hence prove that

$$\int \frac{\cos^5 x}{\sin^4 x} dx = -\frac{\cos^4 x}{3\sin^3 x} + \frac{4\cos^2 x}{3\sin x} + \frac{8\sin x}{3}$$

(c) Show that the are of the upper half of the cardioide $r = a(1-\cos\theta)$ is bisected at $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is 8a. 5

Unit - III

5. Answer any *three* questions : 3×2=6

(a) Find the values of 'a' for which the plane $x+y+z=a\sqrt{3}$ touches the sphere

$$x^2+y^2+z^2-2x-2y-6=0$$

(b) Write down reflection properties of parabola and hyperbola.

(c) Find the equation of the cylinder generated by straight lines parallel to z-axis and passing

through the curve of intersection of the plane $4x+3y-2z=5$ and the surface $3x^2-y^2+2z^2=1$.

(d) Find the equation of the sphere which passes through the circle $x^2+y^2=4, z=0$ and is cut by the plane $x+2y+2z=0$, in a circle of radius 3.

(e) Show that the equation $4xy - 3x^2 = 1$ is transformed to $X^2-4Y^2=1$ by rotating the axes through an angle $\tan^{-1}2$. 2

6. Answer any *one* question : 1×5=5

(a) The expression $ax^2+2hxy+by^2+2gx+2fy+c$ changed to $a'x'^2+2h'x'y'+b'y'^2+2g'x'+2f'y'+c'$ when the axes are rotated through an angle θ. Show that $a+b=a'+b'$. 5

(b) Show that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. 5

[Turn Over]

7. Answer any *one* question : 1×10=10

(a) (i) Prove that the two conics

$$\frac{l_1}{r} = 1 + e_1 \cos\theta \text{ and}$$

$$\frac{l_2}{r} = 1 + e_2 \cos(\theta - \gamma)$$

will touch one another if

$$l_1^2(1-e_2^2) + l_2^2(1-e_1^2) = 2l_1l_2(1-e_1e_2\cos\gamma).$$

5

(ii) A sphere of constant radius r passes through origin O and meet the axes in A, B, C. Prove that the locus of the foot of perpendicular from O to the plane ABC is given by

$$(x^2+y^2+z^2)(x^{-2}+y^{-2}+z^{-2}) = 4r^2 \quad 5$$

(b) (i) Reduce the equation

$$x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$$

to its canonical form and determine the nature of the conic. 5

(ii) Show that the section of the surface

$$yz + zx + xy = a^2$$

by the plane

$$lx + my + nz = p$$

will be a prabola if $\sqrt{l} + \sqrt{m} + \sqrt{n} = 0$. 5

Unit - IV

8. Answer any *two* questions : 2×2=4

(a) Find the differential equation of all parabolas having their axes parallel to y axis.

(b) Find an integrating factor of the differential equation $(x^2+y^2+2x)dx + 2ydy = 0$.

(c) Reduce the equation

$$\sin y.\frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$$

into a linear equation. 2

[Turn Over]

9. Answer any *one* question : 1×5=5

(a) Show that $y=\frac{Q}{P}-e^{-\int pdx}\left[\int e^{\int pdx}d\left(\frac{Q}{P}\right)+C\right]$ is a solution of the differential equation $\frac{dy}{dx}+py=Q$, where P and Q are functions of x only.

(b) If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$ is function of x alone, say $f(x)$, then prove that $e^{\int f(x)dx}$ is an integrating factor of $Mdx+Ndy=0$.