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UG/5th Sem/Math(H)/T/19

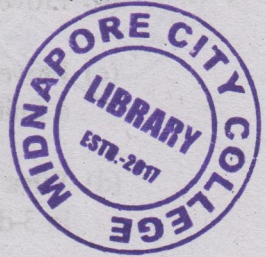
2019

B.Sc. (Honours)

5th Semester Examination

**MATHEMATICS**

Paper - DSE-1T



Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.  
Candidates are required to give their answers  
in their own words as far as practicable.*

**(LINEAR PROGRAMMING)**

**Unit - I**

**(Simplex Algorithm)**

1. Answer any *five* from the following : 2×5

(a) Find a basic feasible solution of the system of equations :

$$x_1 + 2x_2 + 3x_3 = 6$$

$$2x_1 + x_2 + 4x_3 = 4$$

Is the solution degenerate ?

[ Turn Over ]

( 2 )

- (b) Prove that a hyperplane is a convex set.
- (c) Prove that the set of all convex combination of a finite number of points is a convex set.
- (d) What is simplex ? Give an example of a simplex in 3-dimension.
- (e) What is the basic principle of two phase method?
- (f) Define convex polyhedron.
- (g) Put the following LPP in a standard form

$$\text{Minimize } Z = 3x_1 - 4x_2 - x_3$$

$$\text{Subject to, } x_1 + 3x_2 - 4x_3 \leq 12$$

$$2x_1 - x_2 + x_3 \leq 20$$

$$x_1 - 4x_2 - 5x_3 \geq 5$$

$x_1 \geq 0$ ,  $x_2$  and  $x_3$  are unrestricted in sign.

- (h) (i) State Fundamental theorem of L.P.P.
- (ii) State the sufficient condition for a basic feasible solution  $X_B$  to an L.P.P.

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$$\text{Maximize } Z = C^T X$$

$$\text{Subject to, } AX = b, X \geq 0$$

to be optimal.

2. Answer any *one* from the following : 5×1

- (a) Find the solution of the following L.P.P. graphically

$$\text{Maximize } Z = 5x_1 + 3x_2$$

$$\text{Subject to } 3x_1 + 5x_2 \leq 15; 5x_1 + 2x_2 \leq 10.$$

- (b) A firm manufactures three products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4. respectively for each unit of products.

The firm has two machines and below is the required processing time in minutes for each machine on each product.

Machines X and Y have 2000 and 2500 machine-minutes respectively.

		Product		
		A	B	C
Machines	X	4	3	5
	Y	2	2	4

[ Turn Over ]

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The firm manufactures 100 A's, 200 B's and 50 C's but not more than 150 A's.

Set up a L.P.P. to maximize the profit.

3. Answer any *one* from the following :  $10 \times 1$

(a) Solve using two phase method :

$$\text{Minimize } Z = 3x_1 + 5x_2$$

$$\text{Subject to } x_1 + 2x_2 \geq 8$$

$$3x_1 + 2x_2 \geq 12$$

$$5x_1 + 6x_2 \leq 60, \quad x_1, x_2 \geq 0.$$

(b) Solve by simplex method (penalty method).

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + 2x_2 - x_3 \geq 2$$

$$3x_1 - 4x_2 \leq 3$$

$$x_2 + 3x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

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## Unit - II

### (Duality and Special LPP)

4. Answer any *three* from the following :  $2 \times 3$

(a) State the weak duality theorem of a L.P.P.

(b) Give the comparison between transportation and assignment problem.

(c) Find the dual of the following LPP :

$$\text{Maximize } Z = 6x_1 + 5x_2 + 10x_3$$

$$\text{Subject to } 4x_1 + 5x_2 + 7x_3 \leq 5$$

$$3x_1 + \quad + 7x_3 \leq 10$$

$$2x_1 + x_2 + 8x_3 = 20$$

$$2x_2 + 9x_3 \geq 5$$

$x_1, x_3 \geq 0$  and  $x_2$  is unrestricted in sign.

(d) Prove that dual of dual is primal.

(e) Prove that the number of basic variables in a transportation problem is at most  $(m+n-1)$ .

[ Turn Over ]

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5. Answer any *one* from the following :  $5 \times 1$

(a) If  $x$  be any feasible solution to the primal problem and  $v$  be any feasible solution to the dual problem, then  $cx \leq b'y$ .  $5$

(b) Solve the travelling salesman problem where the entries as given as distance. Find minimum distance.

	A	B	C	D	E
A	-	7	6	8	4
B	7	-	8	5	6
C	6	8	-	9	7
D	8	5	9	-	8
E	4	6	7	8	-

6. Answer any *one* from the following :  $10 \times 1$

(a) Find the dual of the following problem and solve the dual problem. Also find the solution of the primal problem from the dual.

$$\text{Maximize } Z = 6x_1 + 4x_2 + 6x_3 + x_4$$

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(h) How many times the graph of the polynomial  $(x^4 - 4)(x^2 + x + 2)$  will cross  $x$ -axis ?

2. Answer any *one* question :  $5 \times 1 = 5$

(a) If the equation  $x^4 + px^3 + qx^2 + rx + s = 0$  has roots of the form  $\alpha \pm i\alpha$ ,  $\beta \pm i\beta$ , where  $\alpha, \beta$  are real, prove that  $p^2 - 2q = 0$  and  $r^2 - 2qs = 0$ . Hence solve the equation  $x^4 + 6x^3 + 18x^2 + 24x + 16 = 0$ .

(b) If the equation  $f(x) = 0$  has all its roots real, then show that the equation  $ff'' - (f')^2 = 0$  has all its roots imaginary.

### Unit - II

#### (Symmetric Function I)

(Marks : 16)

3. Answer any *three* questions :  $2 \times 3 = 6$

(a) If  $\alpha$  be a special root of the equation  $x^8 - 1 = 0$ , then prove that

[ Turn Over ]

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$$(\alpha+2)(\alpha^2+2)\dots(\alpha^7+2) = \frac{2^8-1}{3}$$

(b) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3+qx+r=0$ , then find the value of

$$\sum \frac{1}{\alpha^2 - \beta\gamma}$$

(c) If the roots of the equation

$$x^3+ax^2+bx+c=0$$

are in G.P. then prove that  $b^3 = a^3c$ .

(d) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3+ax+b=0$ , find  $\sum \alpha^5$ .

(e) Show that all imaginary roots of  $x^7=1$  are special roots.

4. Answer any *one* question : 10×1=10

(a) (i) Solve

$$x^3-12x+8=0$$

by Cardan's method.

(ii) Find the special roots of the equation  $x^9-1=0$ . Deduce that

( 7 )

Subject to  $4x_1 + 5x_2 + 4x_3 + 8x_4 = 21$

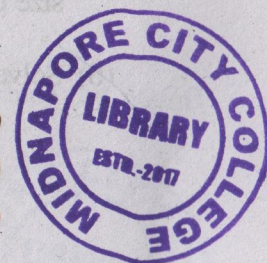
$$3x_1 + 7x_2 + 8x_3 + 2x_4 \leq 48$$

$$x_1, x_2, x_3, x_4 \geq 0$$

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(b) (i) Find the optimal solution of the transportation problem

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	
O <sub>1</sub>	1	2	1	4	30
O <sub>2</sub>	3	3	2	1	50
O <sub>3</sub>	4	2	5	9	20
	20	40	30	10	



using VAM method.

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(ii) Find the optimal assignments for the assignment problem with the following cost matrix

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

4

[ Turn Over ]

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Unit - III

(Game Theory)

7. Answer any two from the following :  $2 \times 2$

(a) State the general dominance rules to reduced the size of pay-off matrix.

(b) Solve the game with the following payoff matrix

		B	
		B <sub>1</sub>	B <sub>2</sub>
A	A <sub>1</sub>	1	3
	A <sub>2</sub>	4	2

(c) State the following terms in concern with the game theory :

Rectangular game, Saddle point, Symmetric game.

8. Answer any two from the following :  $5 \times 2$

(a) Solve the Game graphically :

		B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>
		A <sub>1</sub>	-1	3	2
A <sub>2</sub>		6	2	5	3

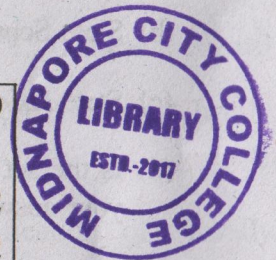
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(b) Transform to LPP and solve the game problem whose payoff matrix is given below, by simplex method.

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

(c) Use dominance property to reduce the payoff matrix and solve the game

0	0	0	0	0	0
4	2	0	2	1	1
4	3	1	3	2	2
4	3	7	-5	1	2
4	3	4	-1	2	2
4	3	3	-2	2	2



[ Turn Over ]

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(POINT SET TOPOLOGY)

Unit - I

(Countable and Uncountable sets)

(Mark - 18)

1. Answer any *four* from the following :  $2 \times 4 = 8$
- (a) Prove that  $\mathbb{N} \times \mathbb{N}$  is countable where  $\mathbb{N}$  denotes the set of all positive integers. 2
- (b) State Schröder-Bernstein Theorem. 2
- (c) State Axiom of choice. 2
- (d) Is the set of all integers well-ordered? Justify your answer. 2
- (e) Define strict partial order on a non-empty set. 2
- (f) State the Zorn's lemma. 2
2. Answer any *two* from the following :  $5 \times 2 = 10$
- (a) Prove that there is no surjective map  $\mathbb{N}$  to the power set of  $\mathbb{N}$  where  $\mathbb{N}$  denotes the set of all positive integers. 5

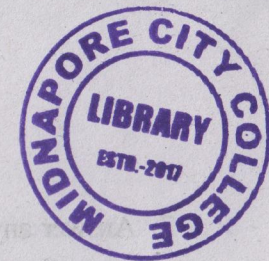
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Unit - III

(Connectednes and Compactness)

(Mark - 21)

6. Answer any *three* from the following :  $2 \times 3 = 6$
- (a) Define a connected topological space. 2
- (b) Define path component of a topological space. 2
- (c) Consider  $(\mathbb{R}, \tau_u)$  where  $\tau_u$  denotes the usual topology on  $\mathbb{R}$ . Is  $(\mathbb{R}, \tau_u)$  compact? Justify your answer. 2
- (d) Give example (with justification) of a locally compact space which is not compact. 2
- (e) Consider  $\mathbb{R}^n$  with the Euclidean metric  $d$ . In the metric topology on  $\mathbb{R}^n$  induced by  $d$ , find the compact subspaces of  $\mathbb{R}^n$ . 2
- (f) "Union of connected subspaces of any topological space is connected" — true or false? Justify your answer. 2



[ Turn Over ]

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7. Answer any *one* from the following :  $5 \times 1 = 5$

(a) Let  $(X, \tau)$  be a topological space and

$\{A_\alpha\}_{\alpha \in J}$  be a family of connected subspaces

of  $X$ . Prove that if  $\bigcap_{\alpha \in J} A_\alpha \neq \phi$  then  $\bigcup_{\alpha \in J} A_\alpha$

is connected. 5

(b) Prove that every closed subspace of a compact space is compact. 5

8. Answer any *one* from the following :  $10 \times 1 = 10$

(a) (i) Prove that product of two connected spaces is connected. 5

(ii) Show that continuous image of a compact space is compact. 5

(b) (i) Define totally bounded metric space.

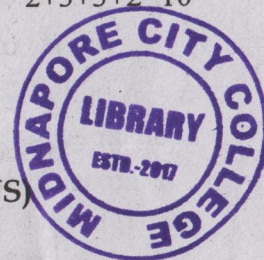
(ii) Prove that if  $(X, d)$  is a totally bounded metric space then  $d$  is a bounded metric on  $X$ .

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(iii) Prove that  $(\mathbb{R}, \bar{d})$  is bounded but not totally bounded where  $\bar{d}$  is a metric defined on  $\mathbb{R}$  by

$$\bar{d}(x, y) = \min\{1, |x - y|\}.$$

(iv) Give example of a metric subspace  $Y$  of  $(\mathbb{R}, d)$  (where  $d(a, b) = |a - b|$  for all  $a, b \in \mathbb{R}$ ) such that  $Y$  is totally bounded but not complete. 2+3+3+2=10



(THEORY OF EQUATIONS)

Unit - I

(Properties of Polynomial Equation)

(Marks : 15)

1. Answer any *five* questions :  $2 \times 5 = 10$

(a) If  $\alpha$  be the imaginary root of the equation  $x^n - 1 = 0$ , where  $n$  is prime, prove that

$$(1 - \alpha)(1 - \alpha^2) \dots (1 - \alpha^{n-1}) = n.$$

[ Turn Over ]



(b) Express  $f(x) = x^4 - 2x^3 - 5x^2 + 10x - 3$  in the form  $(x^2 - x + \lambda)^2 - (ax + b)^2$ .

(c) If  $x^2 + px + 1$  be a factor of  $ax^3 + bx + c$  prove that  $a^2 - c^2 = ab$ .

(d) Apply Descartes rule of signs to find the nature of the roots of the equation  $x^8 + 1 = 0$ .

(e) If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 - 3px^2 + 3qx - 1 = 0$ , then find the centroid of the triangle having vertices

$$\left(\alpha, \frac{1}{\alpha}\right), \left(\beta, \frac{1}{\beta}\right), \left(\gamma, \frac{1}{\gamma}\right).$$

(f) If  $f(x)$  be a polynomial of degree  $n$  then prove that  $f(x) = 0$  cannot have more than  $n$  roots.

(g) If  $1, a, b, \dots, k$  are  $n$  roots of  $x^n - 1 = 0$  then prove that  $(1-a)(1-b)\dots(1-k) = n$ .

(b) (i) Let  $A$  be a set. Define the Cardinal number of  $A$  which is often denoted by  $\text{Card}(A)$ .

2

(ii) Let  $A, B$  be two disjoint sets. If we define the addition of cardinal nos. of  $A$  &  $B$  as follows :

$$\text{Card}(A) + \text{Card}(B) = \text{Card}(A \cup B),$$

then check whether the above definition is well-defined or not.

3

(c) If  $a$  and  $b$  are two real numbers, define  $a < b$  if and only if  $b - a$  is a positive rational number. Show that ' $<$ ' is a strict partial order on  $\mathbb{R}$ . Then exhibit one maximal simply ordered proper subset of  $\mathbb{R}$  (with justification).

2+3=5

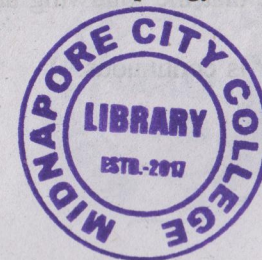
## Unit - II

### (Topological Space)

(Mark - 21)

3. Answer any *three* from the following :  $3 \times 2 = 6$

(a) Define lower limit topology on  $\mathbb{R}$ . 2



[ Turn Over ]

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(b) Let  $X, Y, Z$  be three topological spaces and  $f: X \rightarrow Y, g: Y \rightarrow Z$  be both continuous. Then prove that  $g \circ f$  is a continuous map from  $X$  to  $Z$ . 2

(c) Let  $\tau_1, \tau_2$  be two topologies on a non-empty set  $X$ . Suppose  $\tau_1 \subset \tau_2$ . Let  $A$  be a non-empty subset of  $X$ . Then show that every  $T_2$ -limit point of  $A$  is a  $\tau_1$ -limit point of  $A$ , too. 2

(d) Let  $(X, \tau_1)$  and  $(Y, \tau_2)$  be two topological spaces. Define product topology of  $X$  and  $Y$ . 2

(e) Let  $(X, d)$  be a metric space. The exhibit a basis  $\mathcal{B}$  for the metric topology induced by  $d$  on  $X$ . 2

(f) State Baire Category Theorem. 2

4. Answer any one from the following : 5×1=5

(a) Let  $(X, \tau)$  and  $(Y, \mathcal{U})$  be two topological spaces and  $f: X \rightarrow Y$  be a mapping. Then prove that the following are equivalent.

(i)  $f$  is continuous.

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(ii)  $f^{-1}(S) \in \tau$  for all  $S \in \mathcal{S}$  where  $\mathcal{S}$  is a subbase for  $\mathcal{U}$ . 5

(b) Let  $(X, \tau)$  and  $(Y, \mathcal{U})$  be two topological spaces. Let  $\mathcal{B}$  and  $\mathcal{B}'$  be bases corresponding to  $\tau$  and  $\mathcal{U}$ , respectively. Then prove that

$$\mathcal{D} = \{B \times C \mid B \in \mathcal{B}, C \in \mathcal{B}'\}$$

forms a basis for the product topology on  $X \times Y$ . 5

5. Answer one from the following : 10×1=10

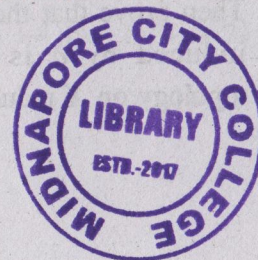
(a) (i) Let  $X = \{a, b, c, d, e\}$ . Verify whether each of the following collections of subsets of  $X$  forms a topology on  $X$  (Give reasons).

$$(1) \tau_1 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$$

$$(2) \tau_2 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$$

$$(3) \tau_3 = \{X, \phi, \{a\}, \{c, d\}, \{a, c, d\}, \{a, b, d, e\}\}.$$

2+2+2=6



[ Turn Over ]

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- (ii) Define finite complement topology on a non-empty set  $X$  and prove that if  $X$  is finite then the finite complement topology on  $X$  is same as the discrete topology on  $X$ .

2+2=4

- (b) (i) Let  $X = A \cup B$  where  $A, B$  are closed in  $(X, \tau)$ . Let  $f: A \rightarrow Y$  and  $g: B \rightarrow Y$  be two continuous functions. If  $f(x) = g(x)$  for all  $x \in A \cap B$ , then prove that  $f$  and  $g$  combine to give a continuous function  $h: X \rightarrow Y$  defined by

$$\begin{aligned} h(x) &= f(x), \text{ when } x \in A \\ &= g(x), \text{ when } x \in B. \end{aligned} \quad 5$$

- (ii) Let  $(X, d)$  be a metric space. Let us consider another metric  $\bar{d}$  on  $X$  defined by

$$\bar{d}(x, y) = \min\{1, d(x, y)\} \text{ for all } x, y \in X.$$

Then prove that the metric topology on  $X$  induced by  $d$  is same as the metric topology on  $X$  induced by  $\bar{d}$ . 5

( 21 )

$2 \cos \frac{2\pi}{9}, 2 \cos \frac{4\pi}{9}, 2 \cos \frac{8\pi}{9}$  are the roots

of the equation  $x^3 - 3x + 1 = 0$ . 4+(2+4)

- (b) (i) Reduce the reciprocal equation

$3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$   
to a reciprocal equation in the standard form and solve it. 5

- (ii) If  $\alpha, \beta, \gamma$  be the roots of the equation

$$x^3 + px^2 + qx + r = 0 (r \neq 0),$$

find the equation whose roots are

$$\frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\gamma}, \frac{1}{\beta} + \frac{1}{\gamma} - \frac{1}{\alpha}, \frac{1}{\alpha} + \frac{1}{\gamma} - \frac{1}{\beta} \quad 5$$

Unit - III

(Symmetric Function II)

(Marks : 14)

5. Answer any two questions : 2×2=4

- (a) Prove that the roots of the equation

$$\begin{aligned} (2x+3)(2x+4)(x-1)(4x-7) + \\ (x+1)(2x-1)(2x-3) = 0 \end{aligned}$$

[ Turn Over ]

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are all real and different. Separate the intervals in which the roots lie.

- (b) Obtain the equation whose roots are the square of the roots of the equation

$$x^4 - x^3 + 2x^2 - x + 1 = 0.$$

- (c) The sum of two roots of the equation

$$x^3 + a_1x^2 + a_2x + a_3 = 0 \text{ is zero, show that } a_1a_2 - a_3 = 0.$$

6. Answer any *two* questions : 5×2=10

- (a) Let

$$f(x) \equiv x^n + p_1x^{n-1} + \dots + p_{n-2}x^2 + p_{n-1}x + p_n = 0$$

be an equation of degree  $n$  having roots

$$\alpha_1, \alpha_2, \dots, \alpha_n. \text{ Let } s_r = \alpha_1^r + \alpha_2^r + \dots + \alpha_n^r,$$

where  $r \geq 0$  is an integer. Then prove that

$$s_r + p_1s_{r-1} + \dots + p_{r-2}s_2 + p_{r-1}s_1 + rp_r = 0,$$

if  $1 \leq r < n$ .

- (b) If  $\alpha, \beta, \gamma, \delta$  be the roots of the equation

$$ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0, \quad a \neq 0 \text{ then prove that}$$

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$$f'(\alpha) + f'(\beta) + f'(\gamma) + f'(\delta) = \frac{32}{a^2} [3abc - a^2d - 2b^3].$$

- (c) If  $\alpha, \beta, \gamma, \delta$  are the roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$ , such that

$$\alpha\beta + \gamma\delta = 0, \text{ then prove that}$$

$$p^2s + r^2 - 4qs = 0.$$

Unit - IV

(Sturm's Theorem)

(Marks : 15)

7. Answer any *one* question : 5×1=5

- (a) Reduce the cubic equation

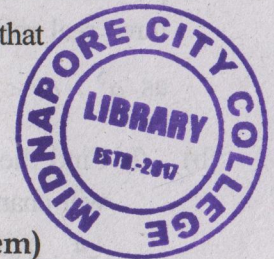
$$ax^3 + 3bx^2 + 3cx + d = 0 \quad (a, b, c, d \text{ are real})$$

to the standard form  $Z^3 + 3HZ + G = 0$  where  $G$  and  $H$  are function of  $a, b, c, d$ . Hence obtain necessary and sufficient condition in terms of  $G$  and  $H$  for the cubic to have two equal roots.

- (b) Find the number of the real roots of the equation

$$x^4 + 4x^3 - x^2 - 2x - 5 = 0 \text{ by using Sturm's method.}$$

[ Turn Over ]



8. Answer any *one* question :

10×1=10

- (a) Define Sturm's functions. Find the Sturm's functions of the polynomial

$$f(x) = x^5 - 5ax + 4b. \text{ If } a \text{ and } b \text{ are positive,}$$

prove that the equation  $x^5 - 5ax + 4b = 0$  has three real roots or only one real root according

$$\text{as } a^5 > \text{ or } < b^4.$$

2+3+5

- (b) (i) Find the transformation  $x = \lambda y + \mu$  which will change the equation

$$x^4 + 4x^3 - 18x^2 - 44x - 7 = 0$$

into reciprocal form. Hence solve the equation.

- (ii) If  $\alpha$  be a multiple root of order 3 of the equation  $x^4 + bx^2 + cx + d = 0$ , show that

$$\alpha = -\frac{8d}{3c}.$$

7+3