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UG/4th Sem/MATH/H/19

2019

B.Sc. (Honours)

4th Semester Examination

**MATHEMATICS**

Paper - C9T

(Multivariate Calculus)

Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers  
in their own words as far as practicable.*

*Illustrate the answers wherever necessary.*

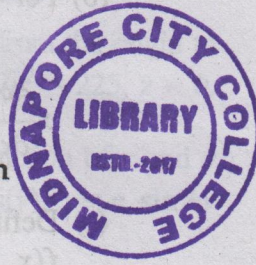
**Unit - I**

1. Answer any *three* questions : 2×3

(a) Show that the limit exists at the origin but the repeated limit does not, for the function

$$f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x}, & xy \neq 0 \\ 0 & , xy = 0 \end{cases}$$

[ Turn Over ]





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(b) For  $F(x, y) = x^4 y^2 \sin^{-1} \frac{y}{x}$  show that

$$x \frac{\partial F}{\partial x} + y \frac{\partial F}{\partial y} = 6F$$

(c) Define directional derivative of the function  $f(x, y)$  at the point  $(a, b)$ . Obtain partial derivative as a special case of it.

(d) Is  $f(x, y) = |y|(1+x)$  differentiable at  $(0, 0)$ ?

(e) Find the maximum or minimum value of

$$f(x, y) = x^3 + y^3 - 3axy.$$

2. Answer any *one* question : 5×1

(a) State and prove sufficient condition for differentiability of a function  $f(x, y)$  at a point  $(a, b)$ .

(b) Let  $(a, b) \in D$ , the domain of definition of  $f$ . If  $f_x(a, b)$  exist and  $f_y(x, y)$  is continuous at  $(a, b)$  then show that  $f(x, y)$  is differentiable at  $(a, b)$ .

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3. Answer any *one* question 10×1

(a) (i) Find the shortest distance from the origin to the hyperbola  $x^2 + 8xy + 7y^2 = 225, z = 0$ .

(ii) If  $z$  be a differentiable function of  $x$  and  $y$  and if  $x = c \cosh(u) \cos(v)$ ,  $y = c \sin hv \sin v$  then prove that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{1}{2}c^2 (\cosh 2u - \cos 2v)$$

$$\left( \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right) \quad 5+5$$

(b) (i) Define total differential of a function  $f(x, y, z)$ .

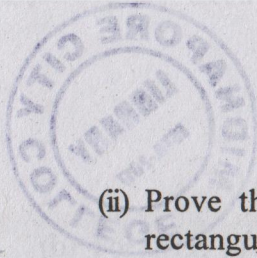
Approximate the change in the hypotenuse of a right angled triangle whose sides are 6 and 8 cm, when the shorter side is

lengthened by  $\left(\frac{1}{4} \text{ cm}\right)$  and the longer is

shortened by  $\left(\frac{1}{8} \text{ cm}\right)$ .

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(ii) Prove that the volume of the greatest rectangular parallelepiped, that can be

inscribed in the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ,

is  $\frac{8abc}{3\sqrt{3}}$ . (2+3)+5

Unit - II

4. Answer any two questions : 2x2

(a) Let

$$f(x, y) = \begin{cases} \frac{1}{2}, & y = \text{rational} \\ x, & y = \text{irrational} \end{cases}$$

verify whether  $\int_0^1 dy \int_0^1 f(x, y) dx$  exists or not.

(b) Evaluate  $\int_0^\infty \frac{\sin rx}{x} dx$  from  $\int_0^\infty \int_0^\infty e^{-xy} \sin rx dx dy$

with the help of change of order of integration.

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(c) Evaluate  $\iint_R (x^2 + y^2) dx dy$  over the region R

bounded by  $xy = 1, y = 0, y = x, x = 2$ .

5. Answer any two questions : 5x2

(a) Show in a diagram the field of integration of the

integral  $\int_0^1 \left( \int_x^{1/x} \frac{y dy}{(1+xy)^2(1+y^2)} \right) dx$  and by

changing the order of integration, show that the

value of the integral is  $\frac{\pi-1}{4}$ .

(b) Are the two iterated integrals  $\int_1^\infty dx \int_1^\infty \frac{x-y}{(x+y)^3} dy$

and  $\int_1^\infty dy \int_1^\infty \frac{x-y}{(x+y)^3} dx$  equal? Justify your answer.

(c) Evaluate

$$\iiint_E \sqrt{a^2 b^2 c^2 - b^2 c^2 x^2 - a^2 c^2 y^2 - a^2 b^2 z^2} dx dy dz$$

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where  $E$  is the region bounded by the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

**Unit - III**

6. Answer any **three** questions : 2×3

(Symbols have their usual meaning)

(a) Find the total work done in moving a particle in a force field given by

$$F = (2x - y + z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y - 5z)\hat{k},$$

along a circle  $C$  in the  $xy$ -plane  $x^2 + y^2 = 9$ ,  $z = 0$ .

(b) Evaluate the vector line integral  $\int_C \vec{F} \times d\vec{x}$  where

$\vec{F} = Z\hat{i}$  and  $C$  is the part of the circular helix  $\vec{x} = b \cos t\hat{i} + b \sin t\hat{j} + c t\hat{k}$  between the points  $(-b, 0, \pi c)$  and  $(b, 0, 0)$ .

(c) Prove that  $\vec{\nabla} \cdot \left[ r\vec{\nabla} \left( \frac{1}{r^3} \right) \right] = 3r - 4$ , where  $\vec{r}$  is

the position vector and  $r = |\vec{r}|$

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(d) Find the equation of the tangent plane to the surface  $xyz = 4$  at the point  $(1, 2, 2)$ .

(e) If  $\Delta\phi = (2xyz^3, x^2z^3, 3x^2yz^2)$  and

$\phi(1, -2, 2) = 4$ , find the function  $\phi$ .

7. Answer any **one** question : 10×1

(a) (i) If  $\vec{\nabla} \cdot \vec{E} = 0, \vec{\nabla} \cdot \vec{H} = 0, \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{H}}{\partial t}$  and

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{E}}{\partial t}, \text{ then show that}$$

$$\nabla^2 \vec{H} = \frac{\partial^2 \vec{H}}{\partial t^2} \text{ and } \nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2}.$$

(ii) Let  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ,  $r = |\vec{r}|$  and  $f(x)$  is a scalar function possessing first and 2nd order derivatives prove that

$$\nabla^2 f(x) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}.$$

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If  $\nabla^2 f(r) = 0$ , show that  $f(r) = A + \frac{B}{r}$

where  $A$  and  $B$  are arbitrary constants.

(b) (i) Prove that

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

(ii) Let  $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$  and  $r = |\vec{r}|$ .

If  $f(r) = \log r$  and  $g(r) = 1/r$ ,  $r \neq 0$ .

Satisfy  $2\nabla f + h(r)\nabla g = 0$  then find  $h(r)$ .

#### Unit - IV

8. Answer any *two* questions :

2×2

(a) Evaluate

$$\int_S (x^2 dy dz + y^2 z dz dx + 2z(xy - x - y) dx dy)$$

where  $S$  is the surface of the cube

$$0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$$

(b) Show that  $\int_S \vec{r} \cdot d\vec{s} = 3v$  where  $v$  is the volume

enclosed by the closed surface  $S$  and  $\vec{r}$  has its usual meaning.

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(c) (i) State Green's theorem in the plane.

(ii) If  $S$  be any closed surface enclosing a volume  $V$  and  $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$ , prove

$$\text{that } \int_S \vec{F} \cdot \hat{n} ds = 6V$$

9. Answer any *one* question :

5×1

(a) Evaluate  $\int_S \vec{F} \cdot \hat{n} dS$ , where

$\vec{F} = x\hat{i} - y\hat{j} + (z^2 - 1)\hat{k}$ ,  $S$  is the surface of the region bounded by  $x^2 + y^2 = 4$ ,  $z = 0$ ,  $z = 4$  in the first octant.

(b) Verify Green's theorem in the plane for  $\oint (xy + y^2) dx + x^2 dy$  where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ .

