Total Pages - 8 UG/5th Sem/Math(H)/T/19

2019

B.Sc. (Honours)

5th Semester Examination

MATHEMATICS

Paper - C11T

(Partial Differential Equations and Applications)

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Unit - I

(1st Order PDE)

1. Answer any *four* questions : 2×4

   (a) Define Quasi-Linear PDE of first order and give an example.

   (b) Form a PDE by eliminating the arbitrary function φ from $Z = e^{ny} \phi(x-y)$.

[Turn Over]

(c) Find the partial differential equation of all spheres of constant radius and having centre on the xy-plane.

(d) Find the characteristic curve of the PDE :

$$yz\frac{\partial z}{\partial x}+xz\frac{\partial z}{\partial y}=xy.$$

(e) Reduce the equation $u_x - xu_y = 0$ in canonical form.

(f) Let $z(x, y)$ be the solution of

$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=4z$ satisfying the condition

$z(x, y)=1$ on the circle $x^2+y^2=1$. Then find

the value of $z(2, 2)$.

2. Answer any *one* question : 5×1

(a) Using the method of separation of variables solve

$$4\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=3z \text{ where } z(0, y)=3e^{-y}-e^{-5y}.$$

(b) Find the integral surface of the linear PDE :

$$x\left(y^2+z\right)\frac{\partial z}{\partial x}-y\left(x^2+z\right)\frac{\partial z}{\partial y}=\left(x^2-y^2\right)z$$

which contains the straight line $x+y=0,\ z=1$.

Unit - II

(2nd Order PDE)

3. Answer any *one* question : 2×1

(a) Determine the region in which the given equation is hyperbolic, parabolic or elliptic

$$u_{xx}+xyu_{xyy}=0.$$

(b) Find the characteristics of the PDE

$$\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}-e^x(2y-3)+e^y=0.$$

4. Answer any *one* question : 10×1

(a) (i) Reduce the equation

$3u_{xx}+10u_{xy}+3u_{yy}=0$ to its canonical form and hence solve it.

[Turn Over]

(ii) Derive the one dimensional wave equation. 6+4

(b) (i) Use the polar co-rodinates r and $\theta(x=r\cos\theta, y=r\sin\theta)$ to transform the Laplace equation $u_{xx}+u_{yy}=0$ into the polar form

$$\nabla^2 u = u_{rr}+\frac{1}{r}u_r+\frac{1}{r^2}u_{\theta\theta}=0$$ 5

(ii) Reduce the Tricomi equation $u_{xx}+xu_{yy}=0$ to the canonical form when $x>0$. 5

Unit - III

(Applications of PDE)

5. Answer any *one* question : 5×1

(a) Find a solution of the following non-homogeneous boundary value problem

$$u_{tt}=c^2u_{xx}+F(x),\ 0<x<l,\ t>0$$

$$u(x, 0)=f(x),\ 0\le x\le l$$

$$u_t(x, 0)=g(x),\ 0\le x\le l$$

$$u(0, t)=A,\ u(l, t)=B,\ t>0.$$ 5

(b) Find the solution of Laplace's equation $\nabla^2\psi=0$ in the semifinite region bounded by $x\ge 0$, $0\le y\le 1$ subject to the boundary conditions

$$\left.\frac{\partial\psi}{\partial x}\right)_{x=0}=0,\ \left.\frac{\partial\psi}{\partial y}\right)_{y=0}=0 \text{ and}$$

$$\psi(x, 1)=f(x).$$ 5

6. Answer any *one* question : 10×1

(a) Obtain the D'Alembert's solution of the Cauchy problem

$$u_{tt}-c^2u_{xx}=0,\quad x\in\mathbb{R},\ t>0$$

$$u(x, 0)=f(x),\quad x\in\mathbb{R}$$

$$u_t(x, 0)=g(x),\quad x\in\mathbb{R}$$

Define the term 'domain of dependence', 'range of influence'. Hence find the solution when

[Turn Over]

$$f(x)=|\sin x|,\ x>0$$
$$=0 \qquad ,\ x<0$$
$$g(x)=0 \qquad ,\ x\in\mathbb{R}$$

(b) Find the temperature distribution of a rod for the following initial boundary value problem

$$u_t = Ku_{xx},\ 0<x<l,\ t>0$$

$$u(0,\ t)=0,\ \ t\ge 0$$

$$u(x,\ 0)=f(x),\ \ 0\le x\le l$$

$$u(l,\ t)=0,\ \ t\ge 0$$

Hence find the solution when $f(x)=x(l-x)$, $0\le x\le l$.

Unit - IV

(Particle Dynamics)

7. Answer any *five* questions : 2×5

(a) Derive the relation $w=\dfrac{v\sin\varphi}{r}=\dfrac{vp}{r}$.

(b) Prove that the acceleration of a particle moving in a plane curve with uniform speed is $\rho\dot{\psi}^2$.

(c) Write the Kepler's Law of planetary motion.

(d) For a particle moving in a central orbit under the inverse square law $\left(\dfrac{\mu}{r^2}\right)$, prove that the velocity (v) at any distance r is given by

$$v^2=\mu\left(\frac{2}{r}-\frac{1}{a}\right).$$

(e) Write the significance of $h=r^2\dot{\theta}$.

(f) Write the differential equation of the central orbit in Pedal form.

(g) A particle describes the parabola $p^2=ar$ under a force, which is always directed towards its focus. Find the law of force.

(h) A point moves along the arc of a cycloid in such a manner that the tangent as it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

[Turn Over]

8. Answer any *two* questions : 5×2

(a) A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.

(b) A machine gun of mass M_o stands on a horizontal plane and contains a shot of total mass m which is fired horizontally at a uniform rate with constant velocity u relative to the gun.

(c) A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection at the end of time $a/\mu V\left(e^{2\pi\mu}-1\right)$ where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction.
