

Total Pages - 8

UG/5th Sem/Math(H)/T/19

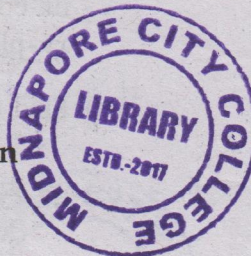
2019

B.Sc. (Honours)

5th Semester Examination

MATHEMATICS

Paper - C11T



(Partial Differential Equations and Applications)

Full Marks : 60

Time : 3 Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

Unit - I

(1st Order PDE)

1. Answer any *four* questions : 2×4
 - (a) Define Quasi-Linear PDE of first order and give an example.
 - (b) Form a PDE by eliminating the arbitrary function ϕ from $Z = e^{my} \phi(x - y)$.

[Turn Over]

(2)

(c) Find the partial differential equation of all spheres of constant radius and having centre on the xy -plane.

(d) Find the characteristic curve of the PDE :

$$yz \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = xy.$$

(e) Reduce the equation $u_x - xu_y = 0$ in canonical form.

(f) Let $z(x, y)$ be the solution of

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 4z \text{ satisfying the condition}$$

$z(x, y) = 1$ on the circle $x^2 + y^2 = 1$. Then find the value of $z(2, 2)$.

2. Answer any *one* question : 5×1

(a) Using the method of separation of variables solve

$$4 \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 3z \text{ where } z(0, y) = 3e^{-y} - e^{-5y}.$$

(3)

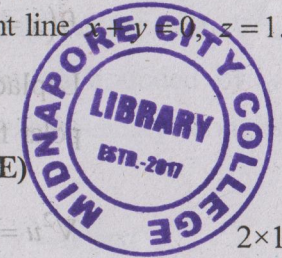
(b) Find the integral surface of the linear PDE :

$$x(y^2 + z) \frac{\partial z}{\partial x} - y(x^2 + z) \frac{\partial z}{\partial y} = (x^2 - y^2)z$$

which contains the straight line $x = y = z = 1$.

Unit - II

(2nd Order PDE)



3. Answer any *one* question : 2×1

(a) Determine the region in which the given equation is hyperbolic, parabolic or elliptic

$$u_{xx} + xyu_{xy} = 0.$$

(b) Find the characteristics of the PDE

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} - e^x(2y - 3) + e^y = 0.$$

4. Answer any *one* question : 10×1

(a) (i) Reduce the equation

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0 \text{ to its canonical form and hence solve it.}$$

[Turn Over]

(4)

(ii) Derive the one dimensional wave equation.

6+4

(b) (i) Use the polar co-ordinates r and

θ ($x = r \cos \theta, y = r \sin \theta$) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into the polar form

$$\nabla^2 u = u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0 \quad 5$$

(ii) Reduce the Tricomi equation

$$u_{xx} + xu_{yy} = 0 \text{ to the canonical form when } x > 0. \quad 5$$

Unit - III

(Applications of PDE)

5. Answer any *one* question : 5×1

(a) Find a solution of the following non-homogeneous boundary value problem

$$u_{tt} = c^2 u_{xx} + F(x), \quad 0 < x < l, \quad t > 0$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l$$

(5)

$$u_t(x, 0) = g(x), \quad 0 \leq x \leq l$$

$$u(0, t) = A, \quad u(l, t) = B, \quad t > 0. \quad 5$$

(b) Find the solution of Laplace's equation $\nabla^2 \psi = 0$ in the semifinite region bounded by $x \geq 0$, $0 \leq y \leq 1$ subject to the boundary conditions

$$\left. \frac{\partial \psi}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial \psi}{\partial y} \right|_{y=0} = 0 \text{ and}$$

$$\psi(x, 1) = f(x). \quad 5$$

6. Answer any *one* question : 10×1

(a) Obtain the D'Alembert's solution of the Cauchy problem

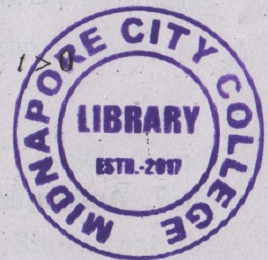
$$u_{tt} - c^2 u_{xx} = 0, \quad x \in \mathbb{R}, \quad t > 0$$

$$u(x, 0) = f(x), \quad x \in \mathbb{R}$$

$$u_t(x, 0) = g(x), \quad x \in \mathbb{R}$$

Define the term 'domain of dependence', 'range of influence'. Hence find the solution when

[Turn Over]



(6)

$$\begin{aligned} f(x) &= |\sin x|, \quad x > 0 \\ &= 0, \quad x < 0 \\ g(x) &= 0, \quad x \in \mathbb{R} \end{aligned}$$

(b) Find the temperature distribution of a rod for the following initial boundary value problem

$$u_t = Ku_{xx}, \quad 0 < x < l, \quad t > 0$$

$$u(0, t) = 0, \quad t \geq 0$$

$$u(x, 0) = f(x), \quad 0 \leq x \leq l$$

$$u(l, t) = 0, \quad t \geq 0$$

Hence find the solution when $f(x) = x(l-x)$,
 $0 \leq x \leq l$.

Unit - IV

(Particle Dynamics)

7. Answer any five questions : 2×5

(a) Derive the relation $w = \frac{v \sin \phi}{r} = \frac{vp}{r}$

(7)

(b) Prove that the acceleration of a particle moving in a plane curve with uniform speed is $\rho\dot{\psi}^2$.

(c) Write the Kepler's Law of planetary motion.

(d) For a particle moving in a central orbit under the inverse square law $\left(\frac{\mu}{r^2}\right)$, prove that the velocity (v) at any distance r is given by

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

(e) Write the significance of $h = r^2\dot{\theta}$.

(f) Write the differential equation of the central orbit in Pedal form.

(g) A particle describes the parabola $p^2 = ar$ under a force, which is always directed towards its focus. Find the law of force.

(h) A point moves along the arc of a cycloid in such a manner that the tangent as it rotates with constant angular velocity. Show that the acceleration of the moving point is constant in magnitude.

[Turn Over]

8. Answer any *two* questions :

5×2

- (a) A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.
- (b) A machine gun of mass M_0 stands on a horizontal plane and contains a shot of total mass m which is fired horizontally at a uniform rate with constant velocity u relative to the gun.
- (c) A particle is projected along the inner surface of a rough sphere and is acted on by no force. Show that it will return to the point of projection at the end of time $a/\mu V(e^{2\pi\mu} - 1)$ where a is the radius of the sphere, V is the velocity of projection and μ is the coefficient of friction.