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UG/4th Sem/MATH/H/19

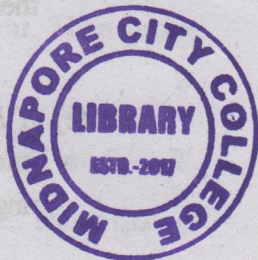
2019

B.Sc. (Honours)

4th Semester Examination

MATHEMATICS

Paper - C10T



Full Marks : 60

Time : 3 Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers

in their own words as far as practicable.

Illustrate the answers wherever necessary.

Unit - 1 (Group A)

1. Answer any **three** questions : 2×3

(a) Prove that in a ring R if a is an idempotent element then $1 - a$ is also idempotent. 2

(b) Define maximal ideal in a Ring. Give it's example. 2

(c) Define char R when char R is called the trivial ring. 2

[Turn Over]

(2)

(d) If a, b be two elements of a field F and $b \neq 0$, then prove that $a = 1$ if $(ab)^2 = ab^2 + bab - b^2$.

2

(e) If R is an integral domain, then $R[x]$ is also an integral domain. Where $R[x]$ is a power series ring.

2

2. Answer any *two* questions : 5×2

(a) Define divisors of zero in a ring. Show that the

ring of matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$ contains

no divisor of zero if $a, b \in Q$ but contains divisor of zero if $a, b \in R$.

5

(b) Show that every field is an integral domain but the converse of the theorem is not necessarily true.

5

(c) Every ideal of the ring of integers $(Z, +, \cdot)$ is a principal ideal.

5

Unit - 2 (Group B)

3. Answer any *two* questions : 2×2

(a) Prove that the rings $(Zn, +, \cdot)$ and $(Z/(n), +, \cdot)$ are isomorphic.

2

(3)

(b) Let $\{R, +, \cdot\}$ and $\{R', +, \cdot\}$ be two rings and $f: R \rightarrow R'$ be a homomorphism. Then prove that $f(-a) = -f(a), \forall a \in R$.

2

(c) Let $R = (Z, +, \cdot); R' = (2Z, +, \cdot)$ and $\phi: R \rightarrow R'$ be defined by $\phi(x) = 2x, x \in Z$, show that ϕ is not a homomorphism.

2

4. Answer any *two* questions : 5×2

(a) State and prove 1st isomorphism theorem of Ring.

5

(b) Let I and J be two ideals of a ring R . Then $I + J$ and $I \cap J$ are also ideals and the factor ring $(I + J)$ and $I/(I \cap J)$ are isomorphic.

5

(c) Let $\{R, +, \cdot\}$ and $\{R', +, \cdot\}$ be two rings and $f: R \rightarrow R'$ be an isomorphism, then prove that

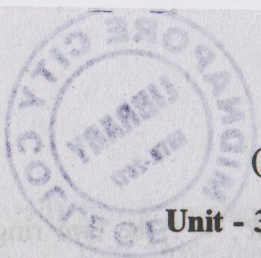
(i) if R be commutative then R' is also Commutative

(ii) if R contains unity then R' also containing unity.

(iii) if R be without divisor of zero then R' is also without divisor of zero.

5

[Turn Over]



(4)

Unit - 3 (Group C)

5. Answer any *two* questions : 2×2

(a) Find the coordinate vector of the vector $(3, -3, 3)$ with respect to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. 2

(b) Find the basis for the subspace

$w = \left[\begin{pmatrix} x & y \\ 0 & t \end{pmatrix} : x+2y+t=0, y+t=0 \right]$ of the vector space of all real 2×2 matrices. 2

(c) Show that the set of real valued discontinuous functions defined on a closed interval does not form a vector space. 2

6. Answer any *one* question : 10×1

(a) (i) Find a basis and dimension of the subspace w in of R^3 where

$$w = \{x, y, z\} \in R^3 : x+2y+z=0, \\ 2x+y+3z=0\}. \quad 4$$

(ii) Show that the set of all R -valued functions defined on $[0, 1]$ having the property $f(x) = f(1-x)$ is a vector space over R . 4

(5)

(iii) If the vectors $(0, 1, a), (1, a, 1), (a, 1, 0)$ of the vector space R^3 over R be linearly dependent, then find the value of a . 2

(b) (i) Suppose $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be a basis of a vector space v over a field F and a nonzero vector β of v is expressed as $\beta = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + c_4\alpha_4 : c_i \in F$

($i = 1, 2, 3, 4$) then if $c_2 \neq 0$ then prove that $\{\alpha_1, \beta, \alpha_3, \alpha_4\}$ is a new basis. 5

(ii) Let A and B be two subspaces of a finite dimensional vector space V . Then $A + B$ is also finite dimensional and

$$\dim(A + B) = \dim A + \dim B - \dim(A \cap B). \quad 5$$

Unit - 4 (Group D)

7. Answer any *three* questions : 2×3

(a) Let $f : R^2 \rightarrow R^2$ be given by

$$f(x, y) = (x^2, y^2 + \sin x). \text{ Then find the linear transformation for the derivative of } f \text{ at } (x, y). \quad 2$$

[Turn Over]

(6)

(b) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation defined by $T(x, y, z) = (x + y, x - z)$.

Then obtain the dimension of the null space of T . 2

(c) If $\phi: v_3 \rightarrow v_1$ and $\phi(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ then show that ϕ is not a linear transformation. 2

(d) Define rank and nullity of a linear transformation. 2

(e) Obtain the matrix of the linear mapping ϕ where

$\phi: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is defined by

$\phi(x, y, z) = (x + y + 2z, 3y - 2z)$. 2

8. Answer any *one* question : 1×10

(a) (i) Let $T: U(F) \rightarrow V(F)$ be a linear transformation and U be finite dimensional then prove that rank of $(T) +$ nullity $(T) = \dim U$. 5

(ii) A matrix of a linear mapping $\phi: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ relative to the order bases $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of \mathbb{R}^3 and $(1, 0), (1, 1)$ of \mathbb{R}^2 is

$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find ϕ . 5

(7)

(b) (i) Prove that a linear transformation $L: v \rightarrow w$ is non-singular if and only if the set $\{Lx_1, Lx_2, \dots, Lx_n\}$ is a basis of w whenever the set $\{x_1, x_2, \dots, x_n\}$ is a basis of v . 4

(ii) Show that the linear operator $V_3(R)$ defined by $T(a, b, c) = (a + b, a - b, 2c)$ is invertible. Find a formula for T^{-1} . 3

(iii) The matrix $m(T)$ of a linear mapping $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ relative to the ordered basis

$((0, 1, 1), (1, 0, 1), (1, 1, 0))$ of \mathbb{R}^3 and

$((1, 0), (1, 1))$ of \mathbb{R}^2 is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$.

Find T . 3

