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2019

B.Sc. (Honours)

4th Semester Examination

MATHEMATICS

Paper - C10T

Full Marks : 60 Time : 3 Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable. Illustrate the answers wherever necessary.

Unit - 1 (Group A)

1. Answer any *three* questions : 2×3

(a) Prove that in a ring R if a is an idempotent element then $1 - a$ is also idempotent. 2

(b) Define maximal ideal in a Ring. Give it's example. 2

(c) Define char R when char R is called the trivial ring. 2

[Turn Over]

(d) If a, b be two elements of a field F and $b \neq 0$, then prove that $a = 1$ if $(ab)^2 = ab^2 + bab - b^2$. 2

(e) If R is an integral domain, then $R[x]$ is also an integral domain. Where $R[x]$ is a power series ring. 2

2. Answer any *two* questions : 5×2

(a) Define divisors of zero in a ring. Show that the ring of matrices of the form $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$ contains no divisor of zero if $a, b \in Q$ but contains divisor of zero if $a, b \in R$. 5

(b) Show that every field is an integral domain but the converse of the theorem is not necessarly true. 5

(c) Every ideal of the ring of integers $(Z, +, .)$ is a principal ideal. 5

Unit - 2 (Group B)

3. Answer any ***two*** questions : 2×2

(a) Prove that the rings $(Zn, +, .)$ and $(Z/(n), +, .)$ are isomorphic. 2

(b) Let $\{R, +, .\}$ and $\{R', +, .\}$ be two rings and $f : R \to R'$ be a homomorphism. Then prove that $f(-a) = -f(a), \forall a \in R$. 2

(c) Let $R = (Z, +, .)$; $R' = (2Z, +, .)$ and $\phi : R \to R'$ be defined by $\phi(x) = 2x, \ x \in Z$, show that ϕ is not a homomorphism. 2

4. Answer any ***two*** questions : 5×2

(a) State and prove 1st isomorphism theorem of Ring. 5

(b) Let I and J be two ideals of a ring R. Then $I + J$ and $I \cap J$ are also ideals and the factor ring $(I + J)$ and $I/(I \cap J)$ are isomorphic. 5

(c) Let $\{R, +, .\}$ and $\{R', +, .\}$ be two rings and $f : R \to R'$ be an isomorphism, then prove that

(i) if R be commutative then R' is also Commutative

(ii) if R contains unity then R' also containing unity.

(iii) if R be without divisor of zero then R' is also without divisor of zero. 5

[Turn Over]

Unit - 3 (Group C)

5. Answer any *two* questions : 2×2

(a) Find the coordinate vector of the vector (3, –3, 3) with respect to the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. 2

(b) Find the basis for the subspace

$$w = \left[\begin{pmatrix} x & y \\ 0 & t \end{pmatrix} : x + 2y + t = 0,\ y + t = 0\right]$$ of the vector space of all real 2 × 2 matrics. 2

(c) Show that the set of real valued discontinuous functions defined on a closed interval does not form a vector space. 2

6. Answer any *one* question : 10×1

(a) (i) Find a basis and dimension of the subspace w in of R^3 where

$$w = \{x, y, z) \in R^3 : x + 2y + z = 0,\ 2x + y + 3z = 0\}.$$ 4

(ii) Show that the set of all R-valued functions defined on [0, 1] having the property $f(x) = f(1 - x)$ is a vector space over R. 4

(iii) If the vectors $(0, 1, a)$, $(1, a, 1)$, $(a, 1, 0)$ of the vector space R^3 over R be linearly dependent, then find the value of a. 2

(b) (i) Suppose $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$ be a basis of a vector space v over a field F and a nonzero vector β of v is expressed as

$$\beta = c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + c_4\alpha_4 : c_i \in F$$

(i = 1, 2, 3, 4) then if $c_2 \neq 0$ then prove that $\{\alpha_1, \beta, \alpha_3, \alpha_4\}$ is a new basis. 5

(ii) Let A and B be two subspaces of a finite dimensional vector space V. Then $A + B$ is also finite dimensional and

$$\dim(A + B) = \dim A + \dim B - \dim(A \cap B).$$ 5

Unit - 4 (Group D)

7. Answer any *three* questions : 2×3

(a) Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x, y) = (x^2, y^2 + \sin x)$. Then find the linear transformation for the derivative of f at (x, y). 2

[Turn Over]

(b) Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y, z) = (x+y, x-z)$.
Then obtain the dimension of the null space of T. 2

(c) If $\phi: v_3 \to v_1$ and $\phi(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2$ then show that ϕ is not a linear transformation. 2

(d) Define rank and nullity of a linear transformation. 2

(e) Obtain the matrix of the linear mapping ϕ where $\phi: R^3 \to R^2$ is defined by

$\phi(x, y, z) = (x+y+2z, 3y-2z)$. 2

8. Answer any ***one*** question : 1×10

(a) (i) Let $T: U(F) \to V(F)$ be a linear transformation and U be finite dimensional then prove that rank of (T) + nullity (T) = dim U. 5

(ii) A matrix of a linear mapping $\phi: R^3 \to R^2$ relative to the order bases (0, 1, 1), (1, 0, 1), (1, 1, 0) of $\mathbb{R}^3$ and (1, 0), (1, 1) of R^2 is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$. Find ϕ. 5

(b) (i) Prove that a linear transformation $L: v \to w$ is non-singular if and only if the set $\{Lx_1, Lx_2, \dots Lx_n\}$ is a basis of w whenever the set $\{x_1, x_2, \dots x_n\}$ is a basis of v. 4

(ii) Show that the linear operator $V_3(R)$ defined by $T(a, b, c) = (a+b, a-b, 2c)$ is invertible. Find a formula for T^{-1}. 3

(iii) The matrix $m(T)$ of a linear mapping $T:\mathbb{R}^3 \to \mathbb{R}^2$ relative to the ordered basis ((0, 1, 1,), (1, 0, 1), (1, 1, 0)) of $\mathbb{R}^3$ and ((1, 0), (1, 1)) of $\mathbb{R}^2$ is $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$.

Find T. 3