2018

CBCS

1st Semester

MATHEMATICS

PAPER—GE1T

(Honours)

Full Marks : 60

Time : 3 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

Calculus Geometry and Differential Equation

Unit—I

1. Answer any *three* questions : 3×2

(a) If $\lim\limits_{x \to 0} \dfrac{ae^{x} + be^{-x} + 2\sin x}{\sin x + x\cos x} = 2$, find the values of a and b.

(b) Draw a rough sketch of $y = \cosh x$. 2

(c) Find the nth derivative of $\frac{1}{x^2 - a^2}$. 2

(d) Find the range of values of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave downwards. 2

(e) From any point P on the parabola $y^2 = 4ax$, perpendiculars PM and PN are drawn to the co-ordinate axes. Find the envelope of the line MN. 2

2. Answer any *one* questions : 10×1

(a) i) Trace the curve $xy^2 = a^2(a - x)$ 5

ii) If $y = (\sin^{-1} x)^2$ prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$$ 5

(b) i) Find the asymptotes of the curve

$$y^3 - yx^2 + y^2 + x^2 - 4 = 0$$

ii) Find if there is any point of inflexion on the curve $y - 3 = 6(x - 2)^5$ 5 + 5

Unit—II

3. Answer any *two* of the following : 2×2

(a) Find the differential of arc length for the curve $x = a(1 - \cos\theta)$, $y = a(\theta + \sin\theta)$.

(b) Find the area of the circle $r = 2a\sin\theta$.

(c) Find the reduction formula for $\int \sec^n x \, dx$.

4. Answer any *two* questions : 2×5

(a) Establish the reduction formula for $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$, m, n being positive integers, greater than 1. Hence Calculate $\int_0^{\pi/2} \sin^5 x \cos^6 x \, dx$.

(b) Find the area bounded by the parabola $4y = 3x^2$ and the straight line $3x - 2y + 12 = 0$.

(c) Find the volume and surface area generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about initial line.

Unit—III

5. Answer any *three* questions : 3×2

(a) Find the angle through which the axes are to be rotated so that the equation $x\sqrt{3} + y + 6 = 0$ may be reduced to the form $x = c$.

(b) If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ *and* $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq = -1$.

(c) Find the the equation of the sphere for which the circle

$x^2 + y^2 + z^2 + 2x - 4y + 2z + 5 = 0$, $x - 2y + 3z + 1 = 0$ is a great circle.

(d) Find the point of intersection of the lines

$r\cos(\theta - \alpha) = p$ and $r\cos(\theta - \beta) = p$

(e) Write down the reflection property of ellipse.

6. Answer any *one* question : 1×5

(a) Show that the distance between two fixed points is unaltered by a rotation of axes.

(b) Find the equation of the cylinder whose generators are parallel to the straight line $2x = y = 3z$ and which passes through the circle $y = 0$, $x^2 + z^2 = 6$.

7. Answer any *one* question : 1×10

(a) i) Prove that the plane $ax + by + cz = 0$ $(a,b,c \neq 0)$ cuts the cone $yz + zx + xy = 0$ in perpendicular straight lines, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$.

ii) Reduce the equation—

$x^2 + 4xy + 4y^2 + 4x + y - 15 = 0$ to its standard form.

5+5

(b) i) Show that the equation of the circle which passes through the focus of the curve $\frac{l}{r} = 1 - e\cos\theta$ and touches it at the point $\theta = \alpha$ is $r(1 - e\cos\alpha)^2 = l\cos(\theta - \alpha) - el\cos(\theta - 2\alpha)$. 5

ii) Prove that the five normals from a given point to a paraboloid lie on a cone. 5

Unit—IV

8. Answer any *two* questions : 2×2

(a) Determine the order and the degree of the differential equation $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x$.

(b) Find an integrating factor of the differential equation $x^2ydx - (x^3 + y^3)\,dy = 0$.

(c) Define singular solution of a differential equation.

9. Answer any *one* question : 1×5

(a) Find a solution of the differential equation $\frac{dy}{dx} - y\tan x = 0$ in the form $y = y_1(x)$. Hence solve $\frac{dy}{dx} - y\tan x = \cos x$ by the substitution $y = y_1(x).v(x)$.

(b) By the substitution $x^2 = u$ and $y^2 = v$ reduce the equation $(px - y)(x - py) = 2p$ to clairant's form and find general solution.

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