2019-20



*Total pages: 3*

**PG (NEW) CBCS**
**M.Sc. Semester-I Examination, 2019**
**PHYSICS**
PAPER: PHS-101

**Full Marks: 40** **Time: 2 Hours**

**Write the answer for each unit in separate sheet**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

**PHS 101.1: METHODS OF MATHEMATICAL PHYSICS - I**

**Marks: 20**

1. **Answer any two questions of the following:** (2× 2 = 4)
   a) If two matrixes A and B can be diagonalized simultaneously, then prove that [A, B]=0
   b) The generating function F(x,t) = $\sum_{n=0}^{\infty} P_n(x)t^n$ for Legendre Polynomial $P_n(x)$ then find the value of $P_3(-1)$.
   c) Prove that $\frac{d}{dx}[\mathrm{erf}(ax)] = -\frac{2a}{\sqrt{\pi}}e^{-a^2x^2}$
   d) If the real part of a complex analytic function f(z) is given as u(x, y)=$e^{-2xy}\sin(x^2-y^2)$, then prove that $f(z) = -ie^{iz^2} + c$
   Where c is a constant.

2. **Answer any two questions of the following:** (4× 2 = 8)
   **a)** Evaluate $\int_{-1}^{+1} P_n(x)P_{n+2}(x)dx$
   **b)** Find the Laurent series of
   $$f(z) = \frac{1}{(z-1)(z-2)}$$
   in the region $1 \le |z| \le 2$ and around z=1
   **c)** Show that the area in the first quadrant enclosed by the curve $(\frac{x}{a})^\alpha + (\frac{y}{b})^\beta = 1, \alpha > 0, \beta > 0$ is given by
   $$\frac{ab}{\alpha+\beta}\cdot\frac{\Gamma\left(\frac{1}{\alpha}\right)\Gamma\left(\frac{1}{\beta}\right)}{\Gamma\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)}$$
   **d)** Using residue theorem, prove that
   $$\int_0^\infty \frac{dx}{(1+x^2)^2} = \frac{\pi}{4}$$

**(Turn over)**

**3. Answer any one question of the following: (8×1=8)**

**a) i)** Find the radius of convergence of the Taylor series expansion of the function $\frac{1}{\cos h(x)}$ around x=0. **(4)**

**ii)** Prove that

$$\iiint_V x^{l-1} y^{m-1} z^{n-1} dxdydz = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n+1)}$$

Where $(x+y+z) \leq 1$. **(4)**

**b) i)**State Jordon's Lemma **(2)**

**ii)** Evaluate $\int_0^\infty \frac{\ln x}{(x^2+1)}$dxby the method of residue. **(6)**

**PHS 101.2: CLASSICAL MECHANICS**
**Marks: 20**

**1. Answer any two questions of the following: (2× 2 = 4)**

**a)** A particle in two-dimension is in a potential V(x,y)=x+2y. Show that the quantity $(P_y-2P_x)$ is a constant of motion.

**b)** Show that the time period of a particle of mass m, undergoing small oscillations around x=0, in the potential $V = V_0 \cosh(\frac{x}{L})$ is $2\pi\sqrt{\frac{mL^2}{V_0}}$

**c)** The potential energy of a particle is given as

$$\phi(x) = x^4 - 4x^3 - 8x^2 + 48x$$

Find out the point at which the particle is of stable and unstable equilibrium.

**d)** Prove that the generating function $F=\sum_k q_k Q_k$ generates the interchange transformation.

**2. Answer any two questions of the following: (4× 2 = 8)**

**a)** In the attractive Kepler problem described by central potential $V(r)=-\frac{k}{r}-\frac{\beta}{r^3}$, one finds that there is a critical value of the angular momentum $L_c$ below which there is no centrifugal barrier. Show that the value of $L_c=(12km^2\beta)^{1/4}$

**b)** Using Poison bracket, show that the transformation

$$Q = (e^{-2q} - p^2)^{1/2}$$
$$P = cos^{-1}(pe^q) \text{ is canonical.}$$

c) A particle of mass m and charge q is moving with velocity $\vec{v}$ in an electric field $\vec{E}$and magnetic field $\vec{B}$then prove that the Lagrangian for the charged particle is

$$L = \frac{1}{2}mv^2 + q(\vec{A}.\vec{v}) - q\phi$$

Where $\phi$ and $\vec{A}$represents scalar a vector potential.

d) State principle of least action. Which quantity is known as Hamilton's characteristic function? Show that the principle of least action in terms of arc length of the particle trajectory may be expressed as

$$\Delta \int \sqrt{2m(H-V)}\, ds = 0$$

**3. Answer any <u>one</u> question of the following:** **(8× 1 = 8)**

a) **I)** Using Hamilton Jacobi method, obtain equation of motion of a particle falling freely under the action of gravity. (Take y direction to be direction of gravity). **(5)**

**II)** Show that, in short wavelength limit, the Schrödinger equation reduces to Hamilton Jacobi equation. **(3)**

b) **I)** Consider two coupled harmonic oscillator of mass m in each. The Hamiltonian describing the operator is

$$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{1}{2}m\omega^2(x_1^2 + x_2^2 + (x_1 - x_2)^2)$$

Compute the normal mode frequencies of such system and hence find out Eigen values of *H*. **(5)**

**II)** Show that Lagrange's bracket remains invariant under canonical transformation. **(3)**

*****