See. 2018-19



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**PG (NEW) CBCS**

**M.Sc. Semester-I Examination, 2018**

**PHYSICS**

PAPER: PHS-101

**Full Marks: 40** **Time: 2 Hours**

**Write the answer for each unit in separate sheet**

The figures in the right-hand margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

**PHS 101.1: METHODS OF MATHEMATICAL PHYSICS - I**

**Marks: 20**

Answer Question number 1, 2 and **any one** between 3 and 4.

1. **Answer any two bits.** (2× 2 = 4)

   a) What is the value of $\alpha$ for which $f(x,y) = 2x + 3(x^2-y^2) + 2i(3xy + \alpha y)$ is an analytic function of complex variable?

   b) Find $\oint \frac{\cos \pi z}{z^2-1} dz$ around a rectangle with vertices $2\pm i$, $-2\pm i$.

   c) If $A=\begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$, show that $(I-A)(I+A)^{-1}$ is a unitary matrix.

   d) Prove that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})}{2}$

2. **Answer any two bits.** (4× 2 = 8)

   a) Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}$

   b) Evaluate $e^A$ and $4^A$ if $\begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}$

   c) Prove that $\int_0^\infty e^{-x^2-2ax}\, dx = \frac{\sqrt{\pi}}{2} e^{a^2}[1 - \text{erf}\,(0)]$

   d) Obtain a set of four orthonormal vectors by Gram–Schmidt method for the vectors

$$\psi_1 = (1,1,0,1)$$
$$\psi_2 = (2,0,0,1)$$
$$\psi_3 = (0,2,3,-2)$$
$$\psi_4 = (1,1,1,-5)$$

3. A) Using Cauchy's integral formula, evaluate the integral

   $I = \oint \frac{z^2}{z^2-1}$ dz around the circle with centre at a) z=1 and b) z= -1. (4)

   B) Prove that

$$\Gamma(m)\Gamma(m+1/2) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$$ (4)

4. A) Prove that the eigen values of a Hermitian matrix are all real and its eigen vectors corresponding to two distinct eigen values are orthogonal. (4)

   B) Evaluate $\oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$, using residue theorem. (4)

*(Turn Over)*

# PHS 101.2: CLASSICAL MECHANICS

## Marks: 20

Answer Question number 1, 2 and **any one** between 3 and 4.



**1. Answer any two bits.** **(2× 2 = 4)**

A) A particle is moving under the action of a generalised potential

$V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$

Find out the magnitude of generalised force.

B) The Lagrangian for a simple pendulum is given by

$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos\theta)$

Find out the Poison Bracket between $\theta$and $\dot{\theta}$.

C) A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. Find out the angular frequency of small oscillation about the minimum of the potential.

D) Let q and p be the Canonical co-ordinate and momentum of a dynamical system. Check whether the following transformations are Canonical or not.

i) $Q_1 = \frac{1}{\sqrt{2}}q^2$ and $P_1 = \frac{1}{\sqrt{2}}p^2$

ii) $Q_2 = \frac{1}{\sqrt{2}}(p+q)$ and $P_2 = \frac{1}{\sqrt{2}}(p-q)$

**2. Answer any two bits.** **(4× 2 = 8)**

a) Show that Lagrange bracket is invariant under canonical transformation.

b) Obtain Hamiltonian and hence equation of motion of a charged particle in an electronic field.

c) Suppose a particle of mass m moves on the frictionless surface of a sphere of radius r under the action of gravity. Using the Lagrange's method of undeterminded multiplier, calculate the critical angle at which the particle flies off the surface.

d) The Lagrangian of a system is given by

$L = \frac{1}{2}m\dot{q}_1{}^2 + 2m\dot{q}_2{}^2 - K[\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2]$

where m and k are positive constants. Find out the frequencies of the normal modes.

**3. Answer any one of the following.**

a) I) Using Hamiltonian-Jacobi method, obtain equation of motion of Harmonic oscillator (1D problem). **(5)**

II) What is action angle variable? How is the introduction of action angle variable simplifies the solution of Harmonic oscillator problem? **(3)**

b) I) A Canonical transformation (q, p) → (Q, P) is made through the generating function $F(q, P) = q^2P$ on the Hamiltonian

$H(p, q) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4}q^4$

where$\alpha$ and $\beta$ are constants. Find out the canonical equations of motion for (Q, P). **(3)**

II) A bead slides on a wire in the shape of a cycloid described by equations

$x = a(\theta - \sin\theta)$

$y = a(\theta + \cos\theta)$

where $0 \leq \theta \leq 2\pi$.

Find a) the Lagrangian function and b) the equation of motion. Neglect fiction between the bead and the wire. **(3)**

III) Show that the transformation from Cartesian to Polar co-ordinate is really possible. **(2)**