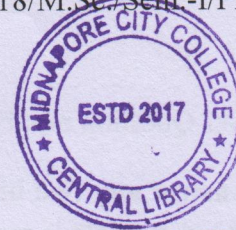


Total page: 2

PG (NEW) CBCS
M.Sc. Semester-I Examination, 2018
PHYSICS
PAPER: PHS-101

**Full Marks: 40****Time: 2 Hours****Write the answer for each unit in separate sheet**

The figures in the right-hand margin indicate full marks.
 Candidates are required to give their answers in their own words as far as practicable.

PHS 101.1: METHODS OF MATHEMATICAL PHYSICS - I**Marks: 20**Answer Question number 1, 2 and **any one** between 3 and 4.**1. Answer any two bits.****(2 × 2 = 4)**

- What is the value of α for which $f(x,y) = 2x + 3(x^2 - y^2) + 2i(3xy + \alpha y)$ is an analytic function of complex variable?
- Find $\oint \frac{\cos \pi z}{z^2 - 1} dz$ around a rectangle with vertices $2 \pm i, -2 \pm i$.
- If $A = \begin{pmatrix} 0 & 1 + 2i \\ -1 + 2i & 0 \end{pmatrix}$, show that $(I - A)(I + A)^{-1}$ is a unitary matrix.
- Prove that $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta = \frac{\Gamma(\frac{1}{4})\Gamma(\frac{3}{4})}{2}$

2. Answer any two bits.**(4 × 2 = 8)**

- Evaluate $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1+x^4}}$
- Evaluate e^A and 4^A if $\begin{pmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{pmatrix}$
- Prove that $\int_0^\infty e^{-x^2 - 2ax} dx = \frac{\sqrt{\pi}}{2} e^{a^2} [1 - \text{erf}(0)]$
- Obtain a set of four orthonormal vectors by Gram-Schmidt method for the vectors

$$\begin{aligned} \psi_1 &= (1, 1, 0, 1) \\ \psi_2 &= (2, 0, 0, 1) \\ \psi_3 &= (0, 2, 3, -2) \\ \psi_4 &= (1, 1, 1, -5) \end{aligned}$$

3. A) Using Cauchy's integral formula, evaluate the integral

$$I = \oint \frac{z^2}{z^2 - 1} dz \text{ around the circle with centre at a) } z=1 \text{ and b) } z=-1. \quad (4)$$

B) Prove that

$$\Gamma(m)\Gamma(m + 1/2) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m) \quad (4)$$

4. A) Prove that the eigen values of a Hermitian matrix are all real and its eigen vectors corresponding to two distinct eigen values are orthogonal. (4)

$$\text{B) Evaluate } \oint \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz, \text{ where } C \text{ is the circle } |z| = 3, \text{ using residue theorem. (4)}$$

(Turn Over)

PHS 101.2: CLASSICAL MECHANICS**Marks: 20**Answer Question number 1, 2 and **any one** between 3 and 4.**1. Answer any two bits.****(2 × 2 = 4)**

A) A particle is moving under the action of a generalised potential

$$V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$$

Find out the magnitude of generalised force.

B) The Lagrangian for a simple pendulum is given by

$$L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos \theta)$$

Find out the Poisson Bracket between θ and $\dot{\theta}$.C) A particle of unit mass moves in a potential $V(x) = ax^2 + \frac{b}{x^2}$, where a and b are positive constants. Find out the angular frequency of small oscillation about the minimum of the potential.

D) Let q and p be the Canonical co-ordinate and momentum of a dynamical system. Check whether the following transformations are Canonical or not.

i) $Q_1 = \frac{1}{\sqrt{2}}q^2$ and $P_1 = \frac{1}{\sqrt{2}}p^2$

ii) $Q_2 = \frac{1}{\sqrt{2}}(p + q)$ and $P_2 = \frac{1}{\sqrt{2}}(p - q)$

2. Answer any two bits.**(4 × 2 = 8)**

a) Show that Lagrange bracket is invariant under canonical transformation.

b) Obtain Hamiltonian and hence equation of motion of a charged particle in an electronic field.

c) Suppose a particle of mass m moves on the frictionless surface of a sphere of radius r under the action of gravity. Using the Lagrange's method of undetermined multiplier, calculate the critical angle at which the particle flies off the surface.

d) The Lagrangian of a system is given by

$$L = \frac{1}{2}m\dot{q}_1^2 + 2m\dot{q}_2^2 - K[\frac{5}{4}q_1^2 + 2q_2^2 - 2q_1q_2]$$

where m and k are positive constants. Find out the frequencies of the normal modes.

3. Answer any one of the following.a) I) Using Hamiltonian-Jacobi method, obtain equation of motion of Harmonic oscillator (1D problem). **(5)**II) What is action angle variable? How is the introduction of action angle variable simplifies the solution of Harmonic oscillator problem? **(3)**b) I) A Canonical transformation (q, p) → (Q, P) is made through the generating function $F(q, P) = q^2P$ on the Hamiltonian

$$H(p, q) = \frac{p^2}{2\alpha q^2} + \frac{\beta}{4}q^4$$

where α and β are constants. Find out the canonical equations of motion for (Q, P). **(3)**

II) A bead slides on a wire in the shape of a cycloid described by equations

$$x = a(\theta - \sin \theta)$$

$$y = a(\theta + \cos \theta)$$

where $0 \leq \theta \leq 2\pi$.Find a) the Lagrangian function and b) the equation of motion. Neglect friction between the bead and the wire. **(3)**III) Show that the transformation from Cartesian to Polar co-ordinate is really possible. **(2)**