Sec. 2

MCC/17/M.Sc./Sem.-1/PHS/1

First Semester Examination-2017

M.Sc. PHYSICS

Paper Code: PHS-101

Full Marks : 40 **Time: 2 Hours**

Use Separate scripts for Group A & Group B

Group A

(Methods of Mathematical Physics)

Answer Question no 1 and any One from the rest.

1. **Answer any five bits.** $2 \times 5 = 10$

(a) Find the value of $\Gamma(-\frac{3}{2})$.

(b) If $f(z) = u(x,y) + iv(x,y)$ is analytic, show that $\vec{\nabla}u.\vec{\nabla}v = 0$.

(c) If a $n \times n$ matrix A has non-degenerate eigen values $\lambda_1, \lambda_2, \ldots, \lambda_n$ then find e^A.

(d) Locate and name the singularity of the function $f(z) = \frac{Sin\sqrt{z}}{\sqrt{z}}$.

(e) Find the inverse of the matrix by Caley-Hamilton theorem $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

(f) Prove that for associated Legendre polynomial $P_n^m(-x) = (-1)^{n+m} P_n^m(x)$.

(g) $P_5(x) = \lambda(63x^5 - 70x^3 + 15x)$. Find the value of λ.

(h) Find an orthonormal basis $W \subset \mathbb{C}^3$ spanned by $w_1 = (1, i, 0)$and $w_2 = (1,2,1-i)$.

2.(a) Prove that $\int_0^\infty \frac{\log(1+x^2)}{(1+x^2)} dx = \pi \log 2$ 4

(b) Prove that $H_{2n}(0) = (-1)^n 2^{2n} (\frac{1}{2})^n$. 3

(c) $A = \begin{pmatrix} 1 & 0 \\ 2 & 4 \end{pmatrix}$. Find the eigen values of $4A^{-1} + 3A = 2I$. 3

3.(a) Consider the vectors $v_1 = (3,0,4), v_2 = (-1,0,7)$ and $v_3 = (2,9,11)$ in $\mathbb{R}^3$. Apply Gram-Schmidt orthogonalization process to obtain an orthonormal set of vectors. 4

(b) $\int_0^\alpha e^{-x^2-2bx} dx = \frac{\sqrt{\pi}}{2} e^{b^2}[1 - \text{erf}(b)]$ 3

(c) Prove that: $xJ_n' = nJ_n - xJ_{n+1}$.

2017-18

Group B

(Classical Mechanics)

Answer Question no 1 and any One from the rest.

1. **Answer any four of the following:** $4 \times 2\frac{1}{2} = 10$

(a) Find Lagrangian of a simple pendulum whose support moves simple harmonically along the vertical line according to $x = a \sin \omega t$.

(b) Using variational principle, prove that the shortest distance between any points in a plane is a straight line.

(c) Using Hamilton's equation of motion, show that the Hamiltonian

$$H = \frac{p^2}{2m} e^{-rt} + \frac{1}{2} m w^2 x^2 e^{rt}$$

leads to the equation of motion of a damped Harmonic oscillator.

(d) Prove that the transformation

$$P = \frac{1}{2}(p^2 + q^2), Q = tan^{-1}\frac{q}{p}$$

is canonical. Find the generating function.

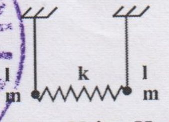
(e) Prove that Poisson's Bracket is invariant under canonical transformation.

(f) Find out the Routhian for the Lagrangian given by

$$L = \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{GmM}{r}$$

where$\mu$is the reduced mass and other symbols have the usual meaning.

2.(a) Two identical simple pendulums are coupled by a spring of constant k. Find the Lagrangian of the system. Obtain Lagrange's equations. $2 + 4$



(b) Using Hamiton- Jacobi theory, derive equation of motion of a simple harmonic oscillator. 4

3.(a) Three particles each of mass m are connected by two identical springs. Find the normal mode frequencies of small oscillations. 5

(b) Derive Lagrange's equation from Hamilton's equation. 5