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PG (NEW) CBCS

M.Sc. Semester-II Examination, 2020

MATHEMATICS

PAPER: MTM-203

Full Marks: 20 **Time: 1 Hour**

Write the answer for each unit in separate sheet

MTM- 203.1

ABSTRACT ALGEBRA

FULL MARK -10

ANSWER ANY ONE QUESTION OF THE FOLLOWING:

1. a) Define conjugacy Relation and Conjugacy class.
   b) Let G be a group and $Z(G)$ be the centre of G, if $a \in Z(G)$ then prove that $cl(a) = \{a\}$ and conversely.
   c) State and prove class equation.
   d) State and prove second isomorphism theorem.
   e) Let $|G| = 255$ then prove that the group G is not simple.
   f) Prove that a finite group of order n is isomorphic to a subgroup of S_n .

MTM-203.2

LINEAR ALGEBRA

FULL MARKS: 10

ANSWER ANY ONE OF THE FOLLOWING QUESTIONS 1 X10

1. For a positive integer n, letP_n denote the space of all polynomials $p(x)$ with coefficients in $\mathbb{R}$ such that $\deg(p(x)) \leq n$ and let B_n denote the standard basis of P_n given by $B_n = \{1, x, x^2, \dots x^n\}$. If $T: P_3 \to P_4$ is the linear transformation defined by $T\big(p(x)\big) = x^2 p'(x) + \int_0^x p(t)$ then find the matrix representation of T with respect to the standard bases $B_3$and B_4.

2. f be a bi-linear form on $\mathbb{R}^2$ such that $f(x_1, x_2) = 2x_1y_1 - 3x_1y_2 + x_2y_2$. Find a matrix A of f in the basis $S = \{u_1 = (1,0), u_2 = (1,1)\}$ as well the matrix B of f in the basis $S' = \{v_1 = (2,1), v_2 = (1,-1)\}$ then find the change of basis matrix P from the basis S to the basis S'.

3. Is $B = \begin{pmatrix} 1 & 3 \\ 2 & -4 \end{pmatrix}$diagonalizable? If yes find P such that $P^{-1}BP$ is a diagonal matrix.

(P.T.O.)

4. (a). Suppose $\{u_1, u_2, ... u_n\}$ is an orthogonal basis for V. Then for any $v \in V$ prove that $v = \frac{<v,u_1>}{<u_1,u_1>}u_1 + \frac{<v,u_2>}{<u_2,u_2>}u_2 + \cdots + \frac{<v,u_2>}{<u_n,u_n>}u_n$.

(b). Let V be the vector space of real continuous function on the interval $-\pi \leq t \leq \pi$ with inner product define by $< f, g >= \int_{-\pi}^{\pi} f(t)dt$. Then, $S = \{1, \sin t, \cos t, \sin 2t\,, \cos 2t\,, ...\}$ is orthogonal or orthonormal?

5. Suppose the characteristic and minimal polynomial of an operator T are respectively $\Delta(t) = (t-2)^4(t-3)^3$ and $m(t) = (t-2)^2(t-3)^2$. Find all possible Jordan Canonical forms with proper explanation.

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