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PG CBCS

M.Sc. Semester-I Examination, 2020

MATHEMATICS

PAPER: MTM 105

(CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS)

Full Marks: 40 **Time: 2 Hours**

Answer any <u>four</u> questions: **10X4=40**

1. Derive the Lagrange's equation of motion for a simple dynamical system. 10

2. a) Show that the Poisson bracket is invariant under the canonical transformation.
   b) State and prove Jacobi's inequality in the context of Poisson bracket. 4+6

3. a) Deduce the Hamilton's equation of motion from the Hamilton's principle.
   b) If the kinetic energy of a dynamical system with n degrees of freedom is a quadratic homogeneous function in the generalised function, then show that the total energy of the system is constant. 5+5

4. A sphere of mass m and of radius a is rolling over a fixed rough sphere of radius b, without slipping. Use Lagrange's equation to find the acceleration of the rolling sphere at any instant of time. Determine the frictional force and the co. efficient of friction. 10

5. Show that with respect to a uniformly rotating reference frame, Newton's second law for a particle of mass m acted upon by real force $\vec{F}$ can be expressed as:

   $\overrightarrow{F_{eff}}=\vec{F}-2m\left(\vec{w}\times\overrightarrow{V_{rot}}\right)-m\vec{w}\times(\vec{w}\times\vec{r})$. Assume that the origins of the inertial and non-inertial coordinate systems are coincident where $\overrightarrow{F_{eff}}$ and $\overrightarrow{V_{rot}}$ represent respectively the effective force and velocity in rotating frame. 10

6. Derive Lorentz transformation equations in special theory of relativity. 10

7. Show that the Coriolis force due to the rotation of earth deflects a vertically falling particle in northern hemisphere toward east and the deflection is proportional to $\sqrt[3]{h}$ for a given latitude where h is the height of the fall. 10

8. Discuss a method to determine the eigen frequencies and normal modes of small oscillation of a dynamical system. 10

[Internal Assessment-10]

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