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PG CBCS

M.Sc. Semester-I Examination, 2020

MATHEMATICS

PAPER: MTM 103

(ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTIONS)

Full Marks: 40 **Time: 2 Hours**

Answer any <u>four</u> questions: **10X4=40**

1. a) Define periodic Strum- Liouville problem.
   b) Construct the green's function for the homogeneous boundary value problem $\frac{d^4y}{dx^4} = 0, y(0) = y'(0) = y'(1) = y(1) = 0.$ 3+7
2. a) All the eigen values of regular SL problem with $r(x) > 0,$ are real.
   b) Define Green's function concerning on ODE. 6+4
3. Write down Confluent hyper geometric equation and find out the series solution on the neighbourhood all possible singularities. 10
4. a) Prove that if $f(z)$ is continuous and has continuous derivatives in [-1, 1] then $f(z)$ has unique Legendre series expansion is given by $f(z) = \sum_{n=0}^{\infty} C_n\, P_n(z)$ where P_n's are Legendre Polynomials$C_{n=} \frac{2n+1}{2} \int_{-1}^{1} f(z)P_n(z)dz$, n=1,2,3…
   b) Prove that $\frac{d}{dz}[J_0(z)] = -J_1(z)$. 7+3
5. a) Deduce the generating function for Legendre's polynomial.
   b) Show that $(n+1)P_{n+1}(z) + nP_{n-1}(z) = (2n+1)zP_n(z)$. 6+4
6. a) Find the general solution of the ODE $2zw''(z) + (1+z)w'(z) - kw = 0$. (where k is a real constant) in series form. Find the value of k for which there is polynomial solution.
   b) Deduce the integral formula for hypergeometric function. 6+4
7. a) Prove that using generating function, $P'_{n+1}(z) - P'_{n-1}(z) = (2n+1)P_n(z)$.
   b) Prove that $1 + 3P_1 + 5P_2 + \cdots + (2n+1)P_n = P'_{n+1} + P'_n\ where\ P_n(z) = P_n$.
   6+4
8. Answer any five questions: 2X5
   a) Find all the singularities of the following differential equation and then classify them:
   $$(z - z^2)\ \ \omega'' + (1-5z)\omega' - 4\omega = 0.$$
   b) Define INDICAL equation in connection with Frobenious method.
   c) Show that $J_n\,(z)$ is an odd function of z if n is odd.
   d) Define orthogonal functions associated with Sturm-Liouville problem.
   e) Under suitable transformation to be considered by you, prove that Legendre differential equation can be reduced to hypergeometric equation.
   f) Prove that:
   $F(-n;b,b;-z) = (1+z)^n$ where F(a;b,c;z) denotes the hypergeometric function.
   g) Show that $\int_{-1}^{1} P_n(z)dz = \begin{cases} 0, n \neq 0 \\ 2, n = 0 \end{cases}$ where the symbol is the usual meaning.
   h) Define a self-adjoint differential equation with an example.

[Internal Assessment-10]

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