

PG CBCS
M.Sc. Semester-I Examination, 2020
MATHEMATICS
PAPER: MTM-101
(REAL ANALYSIS)

Full Marks: 40**Time: 2 Hours****Answer any four questions:****10X4=40**

1. a) Is $GL(n, \mathbb{R})$ is dense in $M(n, \mathbb{R})$?
 b) Prove that \mathbb{R} and \mathbb{R}^2 are not homeomorphic. 5+5
2. a) Show that a metric space X is compact if and only if every real valued continuous function is bounded on X .
 b) Let X and Y be connected metric spaces. Then show that their Cartesian product $X \times Y$ is connected. 5+5
3. State and prove Lebesgue Monotone Convergence theorem and Fatou's Lemma. 10
4. a) Show that continuous image of a connected metric space is connected.
 b) Let X be the union of open disks in \mathbb{R}^2 along with the tangent line $x = 1$. Show that X is connected.
 c) Show that any closed subset of a complete metric space is complete. 4+3+3
5. a) Establish the necessary and sufficient condition for a function $f: [a, b] \rightarrow \mathbb{R}$ to be a function of bounded variation on $[a, b]$.
 b) Check whether the function $f(x) = |x - 1| + |x|$ on $[0, 3]$ is a function of bounded variation or not. If so, also find the variation function of f on $[0, 3]$. 5+5
6. a) Is there exists a non-empty perfect set in \mathbb{R} , having no rational point?
 b) Give an example of non-measurable set with proper justification. 5+5
7. a) Let $f(x) = \frac{1}{9x^7}$ if $0 < x \leq 1$ and $f(0) = 0$. Show that f is Lebesgue integrable on $[0, 1]$ and find the value of the integral.
 b) Evaluate the following: $\int_{-1}^3 7 \sin^3 x \, d(5x^3 + 8[x])$. 7+3
8. Let A and B are non-empty subsets of \mathbb{R} .
 a) If A and B are compact, then prove that $A+B$ is also compact.
 b) If A is closed and B is compact, prove that $A+B$ is closed.
 c) If A and B both are closed, is it always true that $A+B$ is closed? 4+4+2

[Internal Assessment-10]