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PG (NEW) CBCS
M.Sc. Semester-IV Examination, 2020
MATHEMATICS
PAPER: MTM 405B (Special Paper)
(OPERATIONAL RESEARCH MODELLING –II)

Full Marks: 20 **Time: 1 Hours**

Answer any <u>one</u> question of the following: **20 X 1=20**

1. a. Define:

   (i) Reliability of a device.

   (ii) The failure rate of a system.

   b. In a system, there are n number of components connected in parallel with reliability $R_i(t)$; $i = 1,2 \dots, n$. Find the reliability of the system. If $R_1(t) = R_2(t) = \cdots = R_n(t) = e^{-\lambda t}$, λ is the failure rate then find the expression for system reliability.

   c. Show that MTBF of the system of n identical units connected parallelly is

   $$\frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i}.$$

   Where λ is the failure rate of each component.

   d. In a system there are n number of components connected in series with reliability $R_i(t) = n, i = 1,2 \dots, n$. Find the reliability of the system. If $R_i(t) = e^{-\lambda t}$, then find the reliability of the system.

   The system connected in series consists of three independent parts A, B, and C which have MTBF of 100, 400 and 800 hours. Find MTBF of the system and reliability of the system 30 hours. How much MTBF of part A has to be increased to get improvement of MTBF of the system by 30%?

2. a. If the failure distribution has a density function Q(t) and failure rate $\lambda(t)$, show that

   $$1-Q(t)=\exp[-\int_0^t \lambda(t)dt].$$

   b. How many identical components each of which is 90% reliable over a period of 50 hours be used to obtain a 99.99% parallel redundancy system over 50 hours. If we want to obtain the system reliability over a period of 100 hours, how many components should be added?

   c. Prove that the reliability function for random failure is an exponential distribution. How system reliability can be improved?

d. If f(t) is the failure density function of reliability, R(t) is the reliability, Q(t) is the unreliability and Z(t) is the hazard rate, find the relation between
   (i) f(t) and R(t).
   (ii) f(t) and Q(t).
   (iii) Z(t) and R(t).
   (iv) Z(t) and f(t).

3. a. Sate Pontrygin's maximum principle in connection with optimal control problem.
   b. Find the curve X=X(t) which minimizes the functional J=$\int_0^t(\dot{X}^2+1)dt$

      X(0)=1 and X(1)=2.

   c. An electrochemical system is characteristic by the ordinary differential equation $\frac{dx_1}{dt}=x_2$ and $\frac{dx_2}{dt}+x_2=u$, where u is the control variable chosen in a such way that cost functional $\frac{1}{2}\int_0^a(x_1^2+4u^2)dt$ is maximised. Show that, if the boundary conditions are satisfied by the state variables $x_1(0)$ =a, $x_2(0)=b$, where a, b are constants and $x_1 \dashrightarrow 0$, $x_2 \dashrightarrow 0$ as $t \dashrightarrow \infty$ the optimal choice function U is U=-$\frac{1}{2}x_1(t)+(1-\sqrt{2})x_2(t)$.

   d. Let X_m be particular event with probability p_n is divided into m mutually exclusive sub-events $Y_1, Y_2, \ldots, Y_m$ with probability $q_1, q_2, \ldots, q_m$ respectively, such that $p_n=q_1+q_2+\ldots+q_m$ then $H(p_1, p_2, \ldots, p_{n-1}, q_1, q_2, \ldots, q_m)$=$H(p_1, p_2, \ldots, p_{n-1}, p_n)+p_n H(q_1/p_n, q_2/p_n, \ldots, q_m/p_n)$.

4. a. Define entropy function and explain its importance.
   b. Prove that the entropy function $H(p_1, p_2, \ldots, p_n)$ is continuous in p_k $\forall\ 0 \le p_k \le 1$.
   c. Define joint and conditional entropies. Prove that H(X,Y)=H(X/Y)+H(Y)=H(Y/X)+H(X), where H(X)$\geqslant$ H(X/Y).
   d. Discuss the sequencing routes for n jobs in m machines.

5. a. What are MTBF and MTTF in connection with the reliability system.
   b. Suppose a system contains a primary element and a stand-by element. Let λ_p and λ_d present the failure rates of primary element and stand-by element. Find reliability of this system. Also, find the system reliability and MTBF when $\lambda_p=\lambda_d=\lambda$.

c. An industrial process is controlled by a computer and two similar components are operated in stand-by redundancy such that if a computer fails another is instantaneously brought into use in its place. The failure rate of each computer is given by λ=0.01 failure/hour. Compare the improvement in reliability over a single computer when one and then two computers are used in a stand-by. The operating period is 100 hours and the switch is considered to be perfect.

d. Define series and parallel arrangements of reliability of a machinery system with proper example.

e. Derive the process of n jobs through two machines in minimum amount of time.

6. a. What do you mean by memory less channel and noiseless channel?

b. State principle assumption mode on job sequencing problem.

c. Draw a general structure of an information communication system and explain it.

d. Let a car be driven from a stationary position on a horizontal way to a stationary position in garage moving a total distance α. The available controls for the driver are the accelerator and the break. Find the minimum time to bring the car in the stationary position at a distance α when the optimal control to be applied on the car.

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