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PG (NEW) CBCS

M.Sc. Semester-IV Examination, 2020

MATHEMATICS

PAPER: MTM 404B (Special Paper)

NONLINEAR OPTIMIZATION

Full Marks: 40 **Time: 2 Hours**

Answer any <u>one</u> question of the following: **40 X 1=40**

1. a. Solve by using Wolfe's method the following quadratic programming problem

$$Max\ z = 2x_1 + x_2 - x_1^2$$
$$Sub.to\ 2x_1 + 3x_2 \le 6$$
$$2x_1 + x_2 \le 4$$
$$x_1,\ x_2 \ge 0$$

   b. Discuss chance constrained programming technique when only a_{ij} are random variable.

2. a. Find the minimum value of $f(x)$ by geometric programming problem

$$Min\ f(X) = 7x_1x_2^{-1} + 3x_2x_3^{-1} + 5x_1^{-3}x_2x_3 + x_1x_2x_3, x_i \ge 0, i = 1, 2, 3.$$

   b. State and prove Fritz -John saddle point necessary optimality theorem.

3. a. Define Nash equilibrium for a bi matrix games in pure strategy. Prove that all strategically equivalent bi matrix games have the same nash equilibrium.

   b. State and prove Weak duality theorem.

   c. State and prove Motzkin's theorem of alternative.

4. a. Find the Nash equilibrium solution(s) of the following matrix game(if exists)

$$\begin{bmatrix} (-2,\ -1) & (1,\ 1) \\ (-1,\ 2) & (-1,\ -2) \end{bmatrix}.$$

   b. Discuss about the solution procedure of Wolfe's modified simplex method to solve quadratic programming problem

   c. Define quadratic programming problem. Give an example.

5. a. Find geometric-arithmetic mean inequality for a geometric programming problem.

   b. State and prove Slater's theorem of the alternative.

   c. State Farkas' theorem of the non-linear programming and give geometrical interpretation of it.

6. a. State and prove the Fritz- John stationary point necessary optimality theorem.

   b. Write the relationship among the solutions of Local minimization problem (LMP), the minimization problem (MP), the Fritz–John saddle point problem(FJSP), the Fritz-John stationary point problem(FJP), the Kuhn-Tucker stationary point problem(KTP) and the Kuhn-Tucker saddle point problem(KTSP).

7. Answer the questions.
   a. Define posynomial and polynomial in connection with geometric programming with an example.
   b. Let X^0 be an open set in R^n, let θ and g be defined on X^0. Find the conditions under which a solution $(\bar{x}, \bar{r_0}, \bar{r})$ of the Fritz-John saddle point problem is a solution of the Fritz-John stationary point problem and conversely.
   c. Define bi-matrix game with an example.
   d. State Dorn's duality theorem in connection with duality in quadratic programming.
   e. Write the basic difference(s) between Beale's and Wolfe's method for solving quadratic programming problem.
8. a. Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem:
   (i) Complete optimal solution (ii) Pareto optimal solution
   (iii) Local Pareto optimal solution (iv) Weak Pareto optimal solution
   b. Give the geometrical interpretations of differentiable convex function and concave function.
9. a. Let θ be a numerical differentiable function on an open convex set $\Gamma \subset R^n$. θ is convex if and only if $\theta(x^2) - \theta(x^1) \geq \nabla\theta(x^1)(x^2 - x^1)$ for each $x^1, x^2 \in \Gamma$.
   b. Define the following terms:
   (i) The (primal) quadratic minimization problem (QMP).
   (ii) The quadratic dual (maximization) problem (QDP).
10. a. How do you solve the following geometric programming problem?

Find $X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$ that minimizes the objective function

$$f(x) = \sum_{j=1}^{n} U_j(x) = \sum_{j=1}^{N} \left(c_j \prod_{i=1}^{n} x_i^{a_{ij}} \right)$$

$$c_j > 0,\ x_i > 0,\ a_{ij} \quad are\ real\ numbers,\ \forall\ i,\ j$$

   b. Derive the Kuhn-Tucker conditions for quadratic programming problem.

11.a. Solve the following quadratic programming problems by using Beale's method:

Maximize $Z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2$

Subject to the constraints $x_1 + 2x_2 \leq 10$

$x_1 + x_2 \leq 9$

$x_1,\ x_2 \geq 0.$

b. Write short note on complementary slackness principle.

12. a. Prove that a pair $\{y^*,\ z^*\}$ constitutes a mixed strategy Nash equilibrium solution to a Bi-matrix game (A, B) if and only if, there exists a pair $\{p^*,\ q^*\}$ such that $\{y^*,\ z^*,\ p^*, q^*\}$ is a solution of the following bilinear programming problem:

$$\begin{aligned} \text{Minimize} \quad & [y'Az + y'Bz + p + q] \\ \text{subject to} \quad & Az \geq -pl_m \\ & B'y \geq -ql_n \\ & y \geq 0, z \geq 0, y'l_m = 1, z'l_n = 1. \end{aligned}$$

b. Define the following:

(i) Minimization problem;

(ii) Local minimization problem;

(iii) Kuhn-Tucker stationary point problem;

(iv) Fritz-John stationary point problem.

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