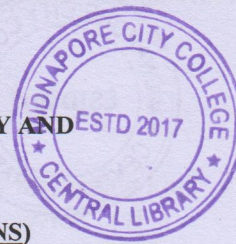


PG (NEW) CBCS
M.Sc. Semester-III Examination, 2019
APPLIED MATHEMATICS WITH OCEANOLOGY AND
COMPUTER PROGRAMMING
PAPER: MTM-302
(TRANSFORMS AND INTEGRAL EQUATIONS)



Full Marks: 40

Time: 2 Hours

1. Answer any four questions of the following:

2×4=8

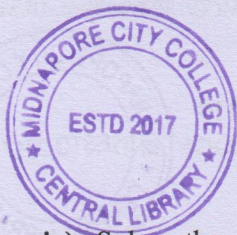
- i) What do you mean by Fredholm integral equation? Give an example for non-homogeneous Fredholm integral equation.
- ii) Define exponential order on Laplace transform? Find the exponential order on the function e^{t^n} ($n > 1$) (if exists).
- iii) Define convolution in Fourier Transform.
- iv) Find the Laplace transform of $f(x) = [x]$, where $[x]$ is the greatest integer function.
- v) Define the wavelet function and explain the parameters involved in it.
- vi) Define the inversion formula for Fourier sine transform of the function $f(x)$. What happens if $f(x)$ is continuous?
- vii) Show that the convolution operator in a Laplace transform is commutative.
- viii) Define singular integral equation with an example.

2. Answer any four questions of the following:

4×4=16

- i) Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ with the initial conditions $y(0) = 1, y'(0) = 0$.
- ii) Solve the following ODE by Laplace transform technique $ty''(t) + 2y'(t) + ty(t) = 0$ with initial conditions $y(0+) = 1, y(\pi) = 0$.
- iii) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}\text{sgn}(\alpha)$, where sgn denotes the Signum function.

(Turn over)



(2)

- iv) Solve the following integral equation $y(x) = f(x) + \lambda \int_{-1}^1 (x+t)y(t)dt$ and the eigen values.
- v) Define the continuous wavelet function and explain the inverse wavelet transform. Write some important applications of wavelets. 1+1+2
- vi) Discuss the solution procedure for solving the homogeneous Fredholm integral equation with separable kernel.
- vii) State and prove convolution theorem of Laplace transform.
- viii) Evaluate $L\{J_0(t)\}$ by the help of initial value theorem.

3. Answer any two questions of the following:

8×2=16

- i) State and prove Parseval's identity on Fourier transform. Use generalization of Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(y^2+b^2)} = \frac{\pi}{ab(a+b)}$, $a, b > 0$.
- ii) the following boundary value problem in the halfplane $y > 0$, described by the PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$, $-\infty < x < \infty$, $y > 0$, with the boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$, u is bounded as $y \rightarrow \infty$; u and $\frac{\partial u}{\partial x}$ both vanish as $|x| \rightarrow \infty$.
- iii) Find the resolvent kernel of the following integral equation and then solve it

$$\varphi(x) = e^{x^2} + \int_0^x e^{x^2-t^2} \varphi(t) dt.$$

If $L\{f(t)\} = F(p)$ which exists $Real(p) > \gamma$ and $H(t)$ is unit step function, then prove that for any α , $L\{H(t-\alpha)f(t-\alpha)\} = e^{-p\alpha}F(p)$ which exists $Real(p) > \gamma$.

5+3=8

(Turn over)

(3)

iv) Solve the homogeneous Fredholm integral equation of the second kind

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t) y(t) dt.$$

Prove that if a kernel is symmetric then all its iterated kernels are also symmetric.

5+3=8

