MIDNAPORE CITY COLLEGE ESTD 2017 CENTRAL LIBRARY

PG (NEW) CBCS

M.Sc. Semester-III Examination, 2019

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER: MTM-302

(TRANSFORMS AND INTEGRAL EQUATIONS)

Full Marks: 40 **Time: 2 Hours**

1. Answer any four questions of the following: **2×4=8**

i) What do you mean by Fredholm integral equation? Give an example for non-homogeneous Fredholm integral equation.

ii) Define exponential order on Laplace transform? Find the exponential order on the function e^{t^n} $(n > 1)$ (if exists).

iii) Define convolution in Fourier Transform.

iv) Find the Laplace transform of $f(x) = [x]$, where $[x]$ is the greatest integer function.

v) Define the wavelet function and explain the parameters involved in it.

vi) Define the inversion formula for Fourier sine transform of the function $f(x)$. What happens if $f(x)$ is continuous?

vii) Show that the convolution operator in a Laplace transform is commutative.

viii) Define singular integral equation with an example.

2. Answer any four questions of the following: **4×4=16**

i) Form an integral equation corresponding to the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$with the initial conditions $y(0) = 1, y'(0) = 0$.

ii) Solve the following ODE by Laplace transform technique $ty''(t) + 2y'(t) + ty(t) = 0$ with initial conditions $y(0+) = 1, y(\pi) = 0$.

iii) Prove that the Fourier transform of $\frac{1}{x}$ is $i\sqrt{\frac{\pi}{2}}sgn(\alpha)$, where sgn denotes the Signum function.

(Turn over)

MIDNAPORE CITY COLLEGE ESTD 2017 CENTRAL LIBRARY

iv) Solve the following integral equation $y(x) = f(x) + \lambda \int_{-1}^{1}(x + t)y(t)dt$ andthe eigen values.

v) Define the continuous wavelet function and explain the inverse wavelet transform. Write some important applications of wavelets. 1+1+2

vi) Discuss the solution procedure for solving the homogeneous Fredholm integral equation with separable kernel.

vii) State and prove convolution theorem of Laplace transform.

viii) Evaluate $L\{J_0(t)\}$ by the help of initial value theorem.

3. **Answer any <u>two</u> questions of the following:** **8×2=16**

i) State and prove Parseval's identity on Fourier transform. Use generalizationof Parseval's relation to show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(y^2+b^2)} = \frac{\pi}{ab(a+b)}, \ a, b > 0.$

ii) the following boundary value problem in the halfplane y>0, describedby the PDE $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0, -\infty < x < \infty, y > 0,$ with the boundary conditions

$u(x, 0) = f(x), -\infty < x < \infty,$ u is bounded as $y \to \infty; u \; and \; \frac{\partial u}{\partial x}$ both vanish as$|x| \to \infty$.

iii) Find the resolvent kernel of the following integral equation and then solve it

$$\varphi(x) = e^{x^2} + \int_0^x e^{x^2 - t^2} \varphi(t)dt.$$

If $L\{f(t)\} = F(p)$ which exists $Real(p) > \gamma$ and H(t) is unit step function, then prove that for any α, $L\{H(t - \alpha)f(t - \alpha)\} = e^{-p\alpha}F(p)$ which exists $Real(p) > \gamma$. **5+3=8**

(Turn over)

iv) Solve the homogeneous Fredholm integral equation of the second kind
$y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt.$

Prove that if a kernel is symmetric then all its iterated kernels are also symmetric. **5+3=8**

****

MIDNAPORE CITY COLLEGE ESTD 2017 CENTRAL LIBRARY