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PG (NEW) CBCS

M.Sc. Semester-III Examination, 2019

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER: MTM- 301

(PARTIAL DIFFERENTIAL EQUATIONS AND GENERALIZED FUNCTIONS)

Full Marks: 40 **Time: 2 Hours**

1. Answer any four questions of the following: **2×4=8**

i) Define Regular and singular distributions.

ii) Using charpit method to solve the following equation $\sqrt{p}+\sqrt{q}=1$.

iii) Show that the Neumann problem for the Poisson's equation has more than one solution.

iv) What are the main differences between an ODE and PDE?

v) Show that there is no maximum principle for the wave equation.

vi) Find the adjoint of the differential operator $L(u)=u_{xx}+u_{tt}-u_t$.

vii) State Basic existence theorem for Cauchy problem.

viii) Solve: $y^2(x-y)p+x^2(y-x)q=z^2(x^2+y^2)$.

2. Answer any four questions of the following: **4×4=16**

i) Show that the type of a linear second order partial differential equation is invariant under a change of coordinates.

ii) Prove that every nonnegative harmonic function in the disk of radius a satisfies $\frac{a-r}{a+r}u(0,0)\leq u(r,\theta)\leq\frac{a+r}{a-r}u(0,0)$.

iii) Solve: $\left(D^2-4D'^2\right)z=\frac{4x}{y^2}-\frac{y}{x^2}$.

iv) State and prove mean value principle for harmonic function.

v) Solve: $\left(x^2D^2-2xyDD'+y^2D'^2-xD+3yD'\right)u=8\frac{y}{x}$. Symbols have their usual meaning.

vi) Define the following with example:

Test function, Convergence of sequence of test function.

(Turn over)

vii) Check the validity of the maximum principle for the harmonic function $\frac{1-x^2-y^2}{1-2x+x^2+y^2}$ in the disk $\bar{D} = \{(x,y): x^2+y^2 \le 1\}$.

viii) Show that the Green's function for laplace equation is symmetric.

3. Answer any <u>two</u> questions of the following: **2×8=16**

i) Show that the Green function for the Laplace equation is symmetric. Establish the Poisson's formula for the solution of a Dirichlet problem for the Laplace equation in a disk of radius a. 3+5

ii) Consider the equation $u_{xx} + 9u_{yy} - u_{xy} = xy^2$. Find a co.ordinate syatem (s,t) in which the equation has the form $9v_{tt} = \frac{1}{3}(s-t)t^2$. Find the general solution u(x,y).

iii) State and prove the strong maximum principle. Establish the laplace equation in polar co.ordinates. 3+5

iv) Using the separation of variables method find a (formal) solution vibrating string with fixed ends.

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < L, 0 < t$$
$$u(0,t) = u(L,t) = 0, \quad t \ge 0$$
$$u(x,0) = f(x), \quad 0 \le x \le L$$
$$u_t(x,0) = g(x), \quad 0 \le x \le L.$$

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