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PG (NEW) CBCS

M.Sc. Semester-II Examination, 2019

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER: MTM-205

(GENERAL THEORY OF CONTINUUM MECHANICS)

Full Marks: 40 **Time: 2 Hours**

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1. Answer any four questions of the following: 2×4

a) Find the relation between α and β such that the small deformation defined by $u_1 = ax_1 + 3x_2$, $u_2 = x_1 - \beta x_2$ and $u_3 = 3x_3$ is isochronic.

b) The components of a stress dyadic at a point referred to the (x_1, x_2, x_3) system are given by

$$(T_{ij}) = \begin{pmatrix} 12 & 9 & 0 \\ 9 & -12 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

For this state of stress, determine the maximum shear stress.

c) Show that the difference between two values of a stream function at two points represents the flux of a fluid across any curve joining these two points.

d) Differentiate between stream line and path line.

e) Show that the extension of a strain quadric in the direction of any central radius vector is equal to the inverse of the square of the radius vector.

f) State the Hooke's law for linear elastic material.

g) Define principal direction and principal strain.

h) Define steady fluid motion.

2. Answer any four questions of the following: 4×4

a) The displacement field for small deformation is given by

$u_1 = (X_1 - X_3)^2$ $u_2 = (X_2 + X_3)^2$ $u_3 = -X_1X_2$

Determine rotation tensor and rotation vector at the point (0, 2, -1).

(Turn over)

b) For the following stress distribution

$$(T_{ij}) = \begin{pmatrix} x^1 + x^2 & T_{12} & 0 \\ T_{12} & x^1 - 2x^2 & 0 \\ 0 & 0 & x^2 \end{pmatrix}$$

Find $T_{12}(x_1, x_2)$ in order that stress distribution is in equilibrium with zero body force and the stress vector on $x_1 = 1$ is given by

$$\vec{T}^n = (1 + x^2)\vec{e}_1 + (2 - x^2)\vec{e}_2$$

c) "The rotation in a fluid can be measured by $\nabla \times \vec{V}$ where $\vec{V}$ be the velocity of the fluid"- Explain it.

d) Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress.

e) Establish the stress vector and stress tensor relationship.

f) Prove that all principal strains are real.

g) Prove that the pressure at a point in a perfect fluid has the same magnitude in every direction.

h) Defining complex potential, fluid the complex potential for a doublet.

3. Answer any two questions of the following: 2×8

a) Derive the maximum shearing strain at any point of the continuum in terms of principal strains at that point.

b) i) Find the integral of the Euler equation of motion for perfect fluid stating necessary assumptions when the flow is rotational and steady.

ii) If $\Phi = (x_1 - t) - (x_2 - t)$ be the velocity potential of a two-dimensional irrotational motion of a continuum, then show that stream lines at time t are $(x_1 - t)^2 - (x_2 - t)^2 = \text{constant}$

c) Derive the basic elastic constants for isotropic elastic solid.

d) State and prove the Kelvins Minimum Energy theorem for a perfect fluid.

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