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PG (NEW) CBCS

M.Sc. Semester-II Examination, 2019

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER: MTM-203

(NUMERICAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

UNIT-1:

MTM 203.1: ABSTRACT ALGEBRA

Marks-20

1. **Answer any two questions of the following:** **2×1**

   a) State the second isomorphism theorem of groups.

   b) Are the groups (R,T) and (R*, .) isomorphic? Justify.

   c) Find all maximal ideals at the ring Z_6.

   d) Define group action with an example.

2. **Answer any two questions of the following:** **4×2**

   a) Show that every group of order p^n where p is a prime, has a nontrivial center and is solvable.

   b) Let R be a a commutative ring with identity and M be an ideal of R. Then M is a maximal ideal of R iff R/M is a field.

   c) Show that $f(x) = x^4-5x^2+x+1$ is irreducible in z[x].

   d) If G is an abelian group having subgroups H_1, H_2, ….., H_t such that $|H_i \cap H_j| = 1$, for all $i \neq j$, then $K = H_1H_2\ldots H_t$ is a subgroup of G of order $|H_1| \times |H_2| \times \ldots\ldots \times |H_t|$ and $K \cong |H1| \times |H2| \times \ldots\ldots \times |Ht|$.

(Turn over)

3. Answer any one questions of the following: 8×1

a) i) State and prove Sylow's first theorem. 4

ii) Show that for each prime p, a finite group G has a Sylow p-subgroup. 4

b) i) Show that if R is a principal ideal domain, then it is a unique factorization domain. Give an example to show that the converse is not true. 5

ii) Let $K \subseteq F$ be a field extension and $\propto \in F$ be algebraic over K. Then show that there exists a unique monic irreducible polynomial $f(x) \in K[x]$ such that $f(\alpha) = 0$. 3

(Turn over)

UNIT-2:

MTM 203.2 : LINEAR ALGEBRA

Marks-20

4. Answer any two questions of the following: **2×1**

a) Define inner product space and also give an example.

b) Let T:V→V be a linear operator. Is it true that $K_{er}T \subseteq K_{er}T^2$? Justify.

c) What do you mean by a normal operator? Are all self-adjoint operators normal? Justify.

d) What is a quotient space in linear algebra?

5. Answer any two questions of the following: **4×2**

a) Determine the linear mapping T : $R^3 \rightarrow R^2$ which maps the basis vectors (1,0,0), (0,1,0), (0,0,1) of R^3 to the vectors (1,1), (2,3), (3,2) respectively. Further (i) find T (1,1,0), T (6,0,-1) (ii) Find $K_{er}T$ and I_mT (iii) prove also that T is not 1-1 but onto.

b) Extend the set of vectors {(2,3,-1), (1,-2,-4)} to an orthogonal basis of the Euclidean space R3 with standard inner product and then find the associated orthonomal basis.

c) Let T be a linear operator on a finite dimensional vector space V. then prove that T is diagonalizable if the minimal polynomial of T is of the form P(t) = $(t - \lambda_1)(t - \lambda_2)\ldots\ldots(t - \lambda_k)$ where $\lambda_1, \lambda_2, \ldots\ldots \lambda_k$ are the distinct eigen values of T.

(Turn over)

d) Let T : $R^3 \to R$ be linear. Show that there exist scalars a, b and c such that T(x,y,z)= ax+by+cz, for all (x,y,z) $\in R^3$. Can you generalize this result for $T:F_n \to F$? Justify your answer.

6. Answer any one questions of the following: 8×1

a) i) Which of the following matrices have Jordan canonical form equal to $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$?

u) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ v) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ 5

w) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ x) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

ii) Let v and w be finite dimensional vector spaces over a field F with dim v=n and w=m. Then prove that the dimension of the linear space L(v,m)= mn. 3

b) i) If A={(2,1), (3,1)} be the basis of $R^2(R)$, then find the dual basis of A.

ii) State and prove first Isomorphism theorem on linear algebra.

iii) Define the characteristic value and characteristics vector of a linear operator on vector space. 3+3+2

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