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PG (NEW) CBCS

M.Sc. Semester-II Examination, 2019

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER: MTM-202

(NUMERICAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

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1. **Answer any four questions of the following:** **2×4**

   a) Define i) natural spline and ii) clamped cubic spline. 1+1

   b) What is Lagrange's bivariate interpolating polynomial?

   c) What is local truncation error for predictor and corrector formula in Milne's predictor- corrector method to solve ordinary differential equation?

   d) What is the advantage of Runge-Kutta method to solve ordinary differential equation over Euler method?

   e) The iterative methods are better than direct methods to solve a system of linear equations. Explain.

   f) What are the advantages to approximate a function using orthogonal polynomials?

   g) To fit a polynomial curve from a table of values, the least square method is better than Taylor's series method with respect to computational time. Justify.

   h) Find the weights w_1, w_2, w_3 so that the relation

   $$\int_{-1}^{1} f(x)dx = w_1 f\left(\sqrt{0.6}\right) + w_2 f(0) + w_3 f(\sqrt{0.6})$$

   is exact for the functions $\int(x) = 1, x, x^2$

2. **Answer any four questions of the following:** **4×4**

   a) Explain how one can solve a system of linear equations using relaxation method.

(Turn over)

b) Given $y' = x^2+y^2$ with $x=0$, $y=1$. Find $y(0.1)$ by fourth order Runge-Kutta method.

c) Solve the following boundary value problem

$y'' + xy' + 1 = 0$

with boundary conditions $y(0)=0$, $y(1)=0$ using finite difference method.

d) Deduce 3-point Gauss-Legendre quadrature formula. What is the order of truncation error of this method?

e) Given

$$f(x) = \begin{cases} x^3 + a_1x^2 + b_1x + c_1, & 0 \le x \le 1 \\ x^3 + a_2x^2 + b_2x + c_2, & 1 \le x \le 2 \end{cases}$$

Find the values of a_1, b_1, c_1, a_2 b_2, c_2 or find the relation among them such that f(x) is a cubic spline.

f) Let $|\lambda_1| > |\lambda_2| > \ldots\ldots|\lambda_n|$ be n eigenvalues of a square matrix A of order $n \times n$. explain how power method helps you to find the eigenvalues λ_n.

g) Find the least squares solution of the system of equations $x+y = 3.0$; $2x-y= 0.03$, $x+3y=7.03$ and $3x+y=4.97$.

h) Define Chebyshev polynomial. Show that it is even under certain conditions to be started by you. Express x^4 in terms of Chebyshev polynomials.

3. Answer any two questions of the following: 2×8

a) Find the root of the following equation using the Bairstow method

$x^4+4x^3-7x^2-22x+24=0$

b) Use the Crank-Nilcolson method to calculate a numerical solution of the problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

(Turn over)

$0 < x < 1$, t>0, where u(0,t)=u(1,t)=0, t>0, u(x,0)=2x, t=0.mention the value of $u(\frac{1}{2},\frac{1}{8})$ by taking $h = \frac{1}{2}$ and $k = \frac{1}{8}$.

c) Describe LU-decomposition method to solve a system of linear equations.

d) Discuss Gauss-Jordon method to find the inverse of a square matrix of order *n* using partial pivoting.

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