PG (NEW) CBCS
M.Sc. Semester-I Examination, 2019
MATHEMATICS
PAPER: MTM-105
(CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS)

Full Marks: 40 **Time: 2 Hours**

1. Answer any four questions of the following. **$4 \times 2 = 8$**

a) Define Poisson bracket.
b) What do you meant by principle of least action?
c) What do you meant by ignorable Coordinate?
d) What are the advantages of Hamiltonian and Lagrangian?
e) State Brachistochrome problem.
f) What do you meant by generating function of a canonical transformation?
g) What do you mean by bifurcation?
h) Define stable fixed point of a dynamical system.

2. Answer any four questions of the following. **$4 \times 4 = 16$**

a) Derive the Lagrangian equation of motion for conservative system.
b) Using Hamilton's equations, find the equations of motion of a projectile in space.
c) In a dynamical system, the kinetic energy and potential energy are given by $T = \frac{1}{2}\frac{\dot{q}_1^2}{a+bq_2^2} + \frac{1}{2}\dot{q}_2^2$ *and* $V = c + dq_2^2$ where a, b, c, d are constants. Find q_1 and q_2.
d) Investigate the stability of the dynamical system $\dot{x} = -(x-2y)(1-x^2-3y^2)$, $\dot{y} = -(x+y)(1-x^2-3y^2)$ at origin $(0,0)$.
e) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.
f) Find the equation of motion of a rigid body rotating with angular velocity ω about a fixed point.
g) Prove that the transformation $Q = \log(\frac{\sin p}{q})$ and $P = qcotp$ is canonical.
h) Prove that, the Poisson bracket of two constant of motion is itself a constant of motion.

3. Answer any two questions of the following. **$8 \times 2 = 16$**

a) If X, Y, Z are three dynamical variables, then prove the Jacobi identity
$$[X,[Y,Z]] + [Y,[Z,X]] + [Z,[X,Y]] = 0.$$
b) State and prove Euler's equations in the context of calculus of variations.
c) Describe the effect of Coriolis force on a particle falling freely under the action of gravity.
d) Describe the saddle node bifurcation for the dynamical system $\dot{x} = p + x^2,\ p \in \mathbb{R}$

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