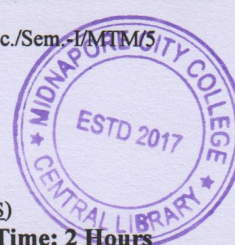


PG (NEW) CBCS
M.Sc. Semester-I Examination, 2019
MATHEMATICS
PAPER: MTM-105

(CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS)



Full Marks: 40

Time: 2 Hours

1. Answer any four questions of the following.

4 × 2 = 8

- a) Define Poisson bracket.
- b) What do you mean by principle of least action?
- c) What do you mean by ignorable Coordinate?
- d) What are the advantages of Hamiltonian and Lagrangian?
- e) State Brachistochrone problem.
- f) What do you mean by generating function of a canonical transformation?
- g) What do you mean by bifurcation?
- h) Define stable fixed point of a dynamical system.

2. Answer any four questions of the following.

4 × 4 = 16

- a) Derive the Lagrangian equation of motion for conservative system.
- b) Using Hamilton's equations, find the equations of motion of a projectile in space.
- c) In a dynamical system, the kinetic energy and potential energy are given by $T = \frac{1}{2} \frac{\dot{q}_1^2}{a+bq_2^2} + \frac{1}{2} \dot{q}_2^2$ and $V = c + dq_2^2$ where a, b, c, d are constants. Find q_1 and q_2 .
- d) Investigate the stability of the dynamical system $\dot{x} = -(x - 2y)(1 - x^2 - 3y^2)$, $\dot{y} = -(x + y)(1 - x^2 - 3y^2)$ at origin (0,0).
- e) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H is the Hamiltonian function.
- f) Find the equation of motion of a rigid body rotating with angular velocity ω about a fixed point.
- g) Prove that the transformation $Q = \log\left(\frac{\sin p}{q}\right)$ and $P = qcot p$ is canonical.
- h) Prove that, the Poisson bracket of two constant of motion is itself a constant of motion.

3. Answer any two questions of the following.

8 × 2 = 16

- a) If X, Y, Z are three dynamical variables, then prove the Jacobi identity $[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0$.
- b) State and prove Euler's equations in the context of calculus of variations.
- c) Describe the effect of Coriolis force on a particle falling freely under the action of gravity.
- d) Describe the saddle node bifurcation for the dynamical system $\dot{x} = p + x^2$, $p \in \mathbb{R}$
