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**PG (NEW) CBCS**

**M.Sc. Semester-I Examination, 2019**

**MATHEMATICS**

PAPER: MTM-102

(COMPLEX ANALYSIS)

**Full Marks: 40** **Time: 2 Hours**

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**1. Answer any four questions of the following:** $4 \times 2 = 8$

a) Find the value of $\oint \frac{cos^2 tz}{z^3} dz$ where C is the circle $|z| = 1$ $and$ $t > 0$.

b) Define branch and branch cut for a multi-valued function f(z).

c) Find the Mobious transformation that maps $0, 1, \infty$ to the respective points $0, i, \infty$.

d) Find the points at which $w = \sin(z)$ is not conformal.

e) Discuss the nature of singularities of the function $f(z) = \frac{\sin z}{(z-\pi)^2}$.

f) If C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining point (1,1) and (2,3), find the value of $\oint (12z^2 - 4iz)dz$.

g) If $(z) = \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$, verify whether Cauchy-Riemann relation are satisfied at the origin or not.

h) What do you mean by mesomorphic function? Give an example.

**2. Answer any four questions of the following:** $4 \times 4 = 16$

a) When α is a fixed real number, show that the function $f(z) = \sqrt[3]{r}e^{i\theta/3}$ $(r > 0, \alpha < \theta < \alpha + 2\pi)$ has derivative everywhere in its domain of definition.

b) By Cantour integration show that $\int_0^{2\pi} \frac{d\theta}{a+bcos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$.

c) Locate and name all singularities of $f(z) = \frac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$.

d) Show that if v and V are harmonic conjugates of u in a domain D, then v(x,y) and V(x,y) can differ at most by an additive constant.

e) Find the number of zeros of the polynomial $z^4 - 5z + 1$ in the annulus $1 < |z| < 2$.

f) When is a function f(z) said to have a pole of order m at $z_0$? If a function f(z) has a pole of order m at $z_0$, prove that $\frac{1}{f(z)}$ has a zero of order m at $z_0$.

g) Without evaluating, find an upper bound for the absolute value of the integral $\oint e^{\bar{z}^2} dz$ where C: $|z| = 1$, traversed in anti clockwise direction.

*(Turn Over)*

h) Use an anti derivative and evaluate the integral $\int_{-1-i\sqrt{3}}^{1+i\sqrt{3}}\left(\frac{5\pi}{z}+3iz^{i-1}\right)dz$ by taking any path of integration in region $y<\sqrt{3}x$ from $z=-1-\sqrt{3}i$ to z=$1+\sqrt{3}i$, expect for its end points. Use principal branches of the required functions.

**3. Answer any two questions of the following:** $8\times 2=16$

a) I) Using Rouche's theorem show that all the roots of $z^7-5z^3+12=0$ lie between the circles $|z|=1$ and $|z|=2$. (4)
II) State and prove Cauchy integral formula for complex function f(z). (4)

b) I) If f(z) is analytic within and on a closed contour expect at a finite number of poles and is not zero on C then $\frac{1}{2\pi i}\int\frac{f'(z)}{f(z)}=N-P$ where N is *the number of zeros* and P is the number of poles inside C. (4)
II) Given f(z) to be analytic, show that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$. (4)

c) I) *Use the Schwarz-Chritoffel transformation to arrive at the* transformation $w=z^m(0<m<1)$, which maps the half plane $y\geq 0$ onto the wedge $|w|\geq 0, 0\leq \arg w\leq m\pi$ and transforms the point z=1 onto the point w=1. (6)
II) Find the fixed points of the transformation $w=\frac{z-1}{z+1}$ (2)

d) I) A linear transformation with the two distinct fixed points α and β can be put in a form $\frac{w-\alpha}{w-\beta}=k\frac{z-\alpha}{z-\beta}$ where k is constant. Under what value/s of k, the above transformation is elliptic, hyperbolic and loxodromic? (5)
II) Evaluate $\oint\frac{e^z}{z^2(z+1)^3}dz$ where C:$9x^2+4y^2=36$. (3)

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