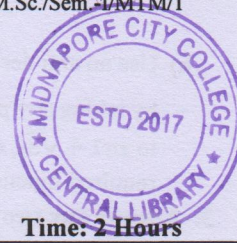


PG (NEW) CBCS
M.Sc. Semester-I Examination, 2019
MATHEMATICS
PAPER: MTM-102
(COMPLEX ANALYSIS)



Full Marks: 40

Time: 2 Hours

1. Answer any four questions of the following:**4 × 2 = 8**

- a) Find the value of $\oint \frac{\cos^2 tz}{z^3} dz$ where C is the circle $|z| = 1$ and $t > 0$.
- b) Define branch and branch cut for a multi-valued function $f(z)$.
- c) Find the Mobious transformation that maps $0, 1, \infty$ to the respective points $0, i, \infty$.
- d) Find the points at which $w = \sin(z)$ is not conformal.
- e) Discuss the nature of singularities of the function $f(z) = \frac{\sin z}{(z-\pi)^2}$.
- f) If C is the curve $y = x^3 - 3x^2 + 4x - 1$ joining point (1,1) and (2,3), find the value of $\oint (12z^2 - 4iz) dz$.
- g) If $f(z) = \begin{cases} \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$, verify whether Cauchy-Riemann relation are satisfied at the origin or not.
- h) What do you mean by mesomorphic function? Give an example.

2. Answer any four questions of the following:**4 × 4 = 16**

- a) When α is a fixed real number, show that the function $f(z) = \sqrt[3]{re^{i\theta/3}}$ ($r > 0, \alpha < \theta < \alpha + 2\pi$) has derivative everywhere in its domain of definition.
- b) By Cantour integration show that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$.
- c) Locate and name all singularities of $f(z) = \frac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$.
- d) Show that if v and V are harmonic conjugates of u in a domain D , then $v(x,y)$ and $V(x,y)$ can differ at most by an additive constant.
- e) Find the number of zeros of the polynomial $z^4 - 5z + 1$ in the annulus $1 < |z| < 2$.
- f) When is a function $f(z)$ said to have a pole of order m at z_0 ? If a function $f(z)$ has a pole of order m at z_0 , prove that $\frac{1}{f(z)}$ has a zero of order m at z_0 .
- g) Without evaluating, find an upper bound for the absolute value of the integral $\oint e^{\bar{z}^2} dz$ where $C: |z| = 1$, traversed in anti clockwise direction.

(Turn Over)

(2)

- h) Use an anti derivative and evaluate the integral $\int_{-1-i\sqrt{3}}^{1+i\sqrt{3}} \left(\frac{5\pi}{z} + 3iz^{i-1} \right) dz$ by taking any path of integration in region $y < \sqrt{3}x$ from $z = -1 - \sqrt{3}i$ to $z = 1 + \sqrt{3}i$, expect for its end points. Use principal branches of the required functions.

3. Answer any two questions of the following:

 $8 \times 2 = 16$

- a) I) Using Rouché's theorem show that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$. (4)
 II) State and prove Cauchy integral formula for complex function $f(z)$. (4)
- b) I) If $f(z)$ is analytic within and on a closed contour expect at a finite number of poles and is not zero on C then $\frac{1}{2\pi i} \int_C \frac{f'(z)}{f(z)} dz = N - P$ where N is the number of zeros and P is the number of poles inside C . (4)
 II) Given $f(z)$ to be analytic, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$. (4)
- c) I) Use the Schwarz-Christoffel transformation to arrive at the transformation $w = z^m$ ($0 < m < 1$), which maps the half plane $y \geq 0$ onto the wedge $|w| \geq 0, 0 \leq \arg w \leq m\pi$ and transforms the point $z=1$ onto the point $w=1$. (6)
 II) Find the fixed points of the transformation $w = \frac{z-1}{z+1}$. (2)
- d) I) A linear transformation with the two distinct fixed points α and β can be put in a form $\frac{w-\alpha}{w-\beta} = k \frac{z-\alpha}{z-\beta}$ where k is constant. Under what value/s of k , the above transformation is elliptic, hyperbolic and loxodromic? (5)
 II) Evaluate $\oint_C \frac{e^z}{z^2(z+1)^3} dz$ where $C: 9x^2 + 4y^2 = 36$. (3)

