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PG (NEW) CBCS

M.Sc. Semester-I Examination, 2019

MATHEMATICS

PAPER: MTM-101

(REAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

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1. **Answer any four questions of the following:** $4 \times 2 = 8$

   a) Define open cover in a metric space. Give an example of an open cover for (0,1) in (R,du).

   b) Define Borel set.

   c) Show that a closed subset of a compact metric space is compact.

   d) For every $\epsilon > 0$ and $f \in L^1(\mu)$, show that $\mu\{x \in X: |f(x)| \geq \epsilon\} \leq \frac{1}{\epsilon}\int f d\mu$.

   e) Show that subtraction of two complex measureable functions on a measurable set X is measurable.

   f) Define σ −algebra with an example.

   g) Evaluate: $\int_{-1}^{1} x^2 d(x^2)$.

   h) Define Lebesgue integral for unbounded measurable function on [a, b].

2. **Answer any four questions of the following:** $4 \times 4 = 16$

   a) State and prove the second Mean-value theorem for Riemann-Stieltjes integral.

   b) Show that every path connected metric space is connected.

   c) Define cantor set. Show that it is uncountable but has measure zero.

   d) State and prove Lebesques's Monotone convergence theorem.

   e) Let $f: [a, b] \to \mathbb{R}$ be a function of bounded variation on [a, b] and let V be its corresponding variation function. Then show that f is continuous at a point $c \in [a, b]$ if and only if V is continuous at c.

   f) Prove that a continuous image of a connected metric space is connected.

   g) Show that the set of all functions of bounded variation on [a, b] forms a vector space under usual addition and multiplication by scalars.

   h) Let $f_n: X \to \mathbb{R}^*$ be measurable for n=1, 2, 3,…..Then show that $\lim sup_{n\to\infty} f_n$ and $sup_{n\to\infty} f_n$ are measurable functions on X.

(Turn Over)

3. Answer any two questions of the following: $2 \times 8 = 16$

i) Let X denotes the set C[0,1] of all real valued continuous functions on [0,1] and we consider the metric d on X given by $d(x,y) = \int_0^1 |x(t) - y(t)|dt \;\; \forall x, y \in X$. Show that the metric space (X,d) is incomplete.

ii) a) Let $f(x) = \frac{1}{7x^{\frac{5}{7}}}$ $if\ 0 < x \leq 1\ and\ f(0) = 0$. Show that f is Lebesque integrable on [0,1] and find the value of the integral. **(5)**

b) Evaluate the following: **(3)**

$$\int_{-1}^{3} 8sin^3x\, d(9x^2 + 5[x]).$$

iii) a) If f(x) is Riemann integrable, in [a,b] then prove that it is also Lebesque integrable there in. Further, give an example of a function which is Lebesque *integrable but not Riemann integrable.* ***(6)***

b) State Egoroff's theorem. **(2)**

iv) a) Show that every path connected metric space is connected. Is the converse true? **(4)**

b) Let μ be a measure on a σ −algebra of subsets of X. Show that the outer measure μ^* induced by μ is countably subadditive. **(4)**

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