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PG (NEW) CBCS
M.Sc. Semester-I Examination, 2018
MATHEMATICS
PAPER: MTM-105
(CLASSICAL MECHANICS AND NON-LINEAR DYNAMICS)

Full Marks: 40 **Time: 2 Hours**

1. Answer any four questions from the following. $4 \times 2 = 8$

a) State basic postulates of special theory of relativity.

b) What do you mean by generalised force? Find the expression of it in terms of generalised co-ordinates.

c) What do you mean by Coriolis force?

d) What do you mean by Euler angle?

e) Find out the relation between variational principle and principle of action.

f) Write a brief note on phase portrait.

g) In conical transformation, what is the relation between old and new Hamiltonian.

h) Define Lyapunov function and Lyapunov stability.

2. Answer any four questions from the following. $4 \times 4 = 16$

a. In the rotating frame system, reduce the relation among Eular force and centrifugal force.

b. A particle is constrained to move on the plane xy = c under the gravity. Obtain Lagrange's equation of motion.

c. Find Lagrangian and its equation of motion for a linear simple harmonic motion.

d. Derive Euler-Lagrangian equation of motion.

e. Show that the transformation

$P = 2(1 + \sqrt{q}\ \text{Cos p})\ \sqrt{q}\ \text{Sin p}$

$Q = \log(1 + \sqrt{q}\ \text{Cos p})$ is canonical and corresponding generating function is $G_3 = -(e^Q - 1)^2 \tan p$.

f. Write a brief note on phase portrait.

g. Construct the Routhian for the two-body problem, for which

$L = \frac{\mu}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$

h. A double pendulum consisting of two masses m_1 and m_2 oscillates in a vertical plane through small angles. m_1 is suspended from a fixed point by light inextensible string of length l_1 and m_2 is suspended from m_1 by a similar string of length l_2.

State the number of degree of freedom of the system and find the equations of motion by using Lagrange'e formulation.

3. Answer any two questions from the following. $8 \times 2 = 16$

a) Derive Hamiltonian-Jacobi equation. Find the Hamiltonian-Jacobi equation of one-dimensional Harmonic oscillator. **(6+2)**

b) What do you mean by bifurcation? Explain Hoft bifurcation of the system

$\ddot{x} + \mu x = 0$ **(4 + 4)**

c) I) A body moves about a point O under no forces. The principle moment of inertia at O being 3A, 5A and 6A. Initially, the angular velocity has components $w_1 = n$, $w_2 = 0$, $w_3 = n$ about the corresponding principle axes. Show that at any time t,

$w_2 = \frac{3n}{\sqrt{5}} \tan h\left(\frac{nt}{\sqrt{5}}\right)$

and that the body ultimately rotates about the mean axis. **(5)**

II) If a transformation from q, p to Q, P be canonical then bilinear form $\sum_i(\delta p_i dq_i - \delta q_i dp_i)$ remains invariant. **(3)**

d) I) Prove that the phase volume is invariant under canonical transformation. **(4)**

II) Show that the Poisson bracket is invariant under canonical transformation. **(4)**

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