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PG (NEW) CBCS
M.Sc. Semester-I Examination, 2018
MATHEMATICS
PAPER: MTM-103
(ORDINARY DIFFERENTIAL EQUATIONS AND SPECIAL FUNCTIONS)

Full Marks: 40 **Time: 2 Hours**

1. **Answer any four questions from the following.** $4 \times 2 = 8$
   a) Define fundamental matrix of system of linear homogeneous differential equation.
   b) Find all the singularities of the following differential equation and then classify them:
   $z^2(z^2-1)^2\omega'' - z(1-z)\omega' + 2\omega = 0$
   c) Define Green's function of the differential operator L of the non-homogeneous differential equation: $Lu(x) = f(x)$.
   d) Under suitable transformation to be considered by you, prove that Legendre differential equation can be reduced to hypergeometric equation.
   e) Prove that:
   $F(-n; b, b; -z) = (1 + z)''$ where $F(a; b; c; z)$ denotes the hypergeometric function.
   f) Consider the boundary value problem
   $\frac{d^2y}{dx^2} + \lambda y = 0, 0 \leq x \leq \pi$
   Subject to $y(0) = 0$, $y(\pi) = 0$. Find the values of λ for which the boundary value problem is solvable.
   g) Establish the integral representation of confluent hypergeomagnetic function.
   h) Find the indicial equation of the Bessel equation
   $$z^2\frac{d^2w}{dz^2} + z\,\frac{dw}{dz} + (z^2 - \,v^2)w = 0$$
   corresponding to its singularity.
2. **Answer any four questions from the following.** $4 \times 4 = 16$
   a) Let $w_1(z)$ and $w_2(z)$ be two solutions of $(1 - z^2)w''(z) - 2zw'(z) + (\sec z)w=0$ with Wronskian $w(z)$. If $w_1(0) = 1$, $w'(0) = 0$ and $w(\frac{1}{2}) = \frac{1}{3}$, then find the value of $w_2'(z)$ at $z = 0$.
   b) Using Greens function method, solve the following differential equation $y'''(x) = 1$, subject to boundary condition $y(0) = y(1) = 0$, $y'(0) = y'(1)$.
   c) Find the generating function for Bessel function of integral order.
   d) Deduce Rodrigue's formula for Legendre's polynomial.
   e) Find the general solution of the homogeneous equation
   $\frac{dx}{dt} = Ax$; where $A = \begin{pmatrix} 3 & 2 \\ 5 & 1 \end{pmatrix}$ and $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.
   f) Show that $1 + 3P_1(z) + 5P_2(z) + 7P_3(z) + \ldots + (2n+1)P_n(z) = \frac{d}{dz}[P_{n+1}(z) + P_n(z)]$ where $P_n(z)$ denotes the Legendre's Polynomial of degree n.

(Turn Over)

g) If the vector function $\phi_{1,}\phi_{2,}\ \ldots, \phi_n$ defined as follows

$$\phi_1 = \begin{bmatrix}\phi_{11}\\ \cdot \\ \phi_{n1}\end{bmatrix}, \phi_2 = \begin{bmatrix}\phi_{21}\\ \cdot \\ \phi_{n2}\end{bmatrix}, \ldots, \phi_n = \begin{bmatrix}\phi_{1n}\\ \cdot \\ \phi_{nn}\end{bmatrix}$$

be n solutions of the homogeneous linear differential equation $\frac{dx}{dt}$ = A(t)x(t) in the interval a $\leq t \ \leq b$, then these n solutions are linearly independent in a $\leq t \ \leq b$ iff Wronskian W$[\phi_1, \phi_2, \ldots, \phi_n] \neq 0$ $\forall$t, on a $\leq t \ \leq b$.

h) Prove that

$\int_{-1}^{1} P_m\,(z)P_n$(z) dz = 0, when m$\neq$ n and $P_m(z)$, $P_n(z)$ are Legendre's polynomial.

3. Answer any two questions from the following **$(2 \times 8 = 16)$**

A) i) Find the characteristics values and characteristic functions of the Sturn-Liouville problem $(x^3y')' + \lambda xy = 0$; y(1) = 0, y(e) = 0 **(4)**

ii) Determine whether the matrix B = $\begin{pmatrix} e^{4t} & 0 & 2e^{4t}\\ 2e^{4t} & 3e^t & 4e^{4t}\\ e^{4t} & e^t & 2e^{4t}\end{pmatrix}$ is a fundamental matrix of the system $\frac{dx}{dt}$ = AX where X = $\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$, A = $\begin{pmatrix}1 & -3 & 9\\ 0 & -5 & 18\\ 0 & -3 & 10\end{pmatrix}$. If not justify, find another alternative fundamental matrix. **(4)**

B) i) How do you solve the homogeneous vector differential equation in the form $\frac{dx}{dt}$ = AX where X = $\begin{pmatrix}x_1\\ \cdot \\ x_n\end{pmatrix}$ and A = $(aij)_{n\times n}$ matrix. [Assuming that the eigen values of A are all real and distinct]. **(6)**

ii) Deduce the confluent hypergeometric differential equation. **(2)**

C) i) Using Green's function, solve the boundary value problem

Y'' – y = x, y(0) = y(1) 0 **(5)**

ii) Prove that

$$J_{\frac{1}{2}}^2(z) + J_{-\frac{1}{2}}^2(z) = \frac{2}{\pi z}$$

Also prove that

$$J_0^1(z) = -J_1\ (z)$$ **(3)**

D) i) Find the general solution of the ODE 2zw''(z) + (1+z)w'(z) – kw = 0.(where k is a real constant) in series form for which values of k is there a polynomial solution? **(5)**

ii) Deduce the integral formula for hypergeometric function. **(3)**

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