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PG (NEW) CBCS

M.Sc. Semester-I Examination, 2018

MATHEMATICS

PAPER: MTM-101

(REAL ANALYSIS)

Full Marks: 40 **Time: 2 Hours**

1. **Answer any four questions from the following.** $\mathbf{4 \times 2 = 8}$

   a) Show that if a Cauchy sequence in a metric space has a convergent subsequence, then the whole sequence is convergent.

   b) Define cover and sub cover in a metric space. Give an open cover of (0, 1).

   c) Is the following a connected subset of R^2

      {(x, y) ∈ R^2: $x^{2/3} + y^{2/3}$ = 1} Justify your answer.

   d) Show that every compact metric space is separable.

   e) If f(x) = $9x^2$-2x and g(x) = 5, then find the value of R-S integral $\int_3^7 f(x)dg(x)$.

   f) Show by an example that closed and bounded subsets of a metric space is not necessarily compact.

   g) Show that countable union of null subsets of ℝ is a null set.

   h) Find the total variation of the function f(x) = 3x + 2 over the interval [2, 5].

2. **Answer any four questions** $\mathbf{4 \times 4 = 16}$

   a) Establish a necessary and sufficient condition for a function f : [a, b] → ℝ to be a function of bounded variation on [a, b].

   b) Check whether the function f(x) = |x - 1| + |x| on [0, 3] is a function of bounded variation or not. If so, also find the variation function of f(x) on [0, 3].

   c) Show that a metric space (x, d) is connected if every continuous functions f : x → {0, 1} is constant, where two point set {0, 1} is considered to be a metric space under discrete metric.

   d) Suppose f is bounded on [a, b], f has finitely many points of discontinuity on [a, b] and α is continuous at every point at which f is discontinuous. Then show that f ∈ $\mathcal{R}(\alpha)$ on [a, b].

   e) Show that continuous image of a connected metric space is connected.

   f) Let the functions f(x) and g(x) defined on [-1, 5] as follows

      f(x) = $\begin{cases} 0, & -1 \le x \le 3 \\ 2x+1, & 3 \le x \le 5 \end{cases}$

      and g(x) = $\begin{cases} 2-3x^2, & -1 \le x \le 3 \\ 6, & 3 \le x \le 5 \end{cases}$

      Show that f(x) is not R-S integrable w.r.t. g(x) on [-1, 5].

   g) State general extension theorem. Let x be a non-empty set and $\varphi \neq E \subseteq$ X and let ξ be a semi algebra of subsets of x. Let $\xi \cap E$ = {A ∩ E : A ∈ ξ}. Show that $\xi \cap E$ is semi algebra if subsets of E.

   h) Define Lebesgue integral for unbounded function. Show that f(x) is Lebesgueintegrable on [0,1] and find the value of the integral $\int_0^1 f(x)dx$

      where f(x) = $\begin{cases} \frac{1}{x^{4/5}}, & 0 < x \le 1 \\ 0, & x = 0 \end{cases}$

(Turn Over)

3. Answer any two questions **$2 \times 8 = 16$**

i) a) Suppose f $\in L^1(\mu)$. Prove that to each $\epsilon > 0$ there exists $\delta > 0$ such that $\int_E |f| d\mu < \grave{o}$ whenever $\mu(E) < \delta$. **(4)**

b) Let $\{E_k\}$ be a sequence of measurable sets in X, such that $\sum_{k=1}^{\propto} \mu(E_k) < \propto$
Then prove that almost all x $\in$ X lie in at most finitely many of the sets E_k. **(4)**

ii) a) Define integrand of non-negative measurable function. State and prove the Lebesgue monotone convergence theorem. **(6)**

b) Evaluate the following: **(2)**

$$\int_{-1}^{3} 2\cos x \, d(2x + [x])$$

iii) Prove that a bounded function f(x) on [a, b] is Lebesgue integrable on [a, b] if for every $\varepsilon > 0 \sqsupset a$ measurable position p of [a, b] such that U(P, f) – L(P, f) <ε

Consider the function f(x) is defined by

$$f(x) = \begin{cases} 3, & when\ x\ is\ rational\ in\ [1,6] \\ 4, & when\ x\ is\ irrational\ in\ [1,6] \end{cases}$$

The f is not Riemann integrable on [1, 6], but it is Lebasgue integrable on [1, 6]. **(8)**

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